Compositional Synthesis of Multi-Robot Motion Plans via SMT Solving

Indranil Saha

UC Berkeley and UPenn

Joint work with

Rattanachai Ramaithitima (UPenn), Vijay Kumar (UPenn), George Pappas (UPenn) and Sanjit Seshia (UC Berkeley)

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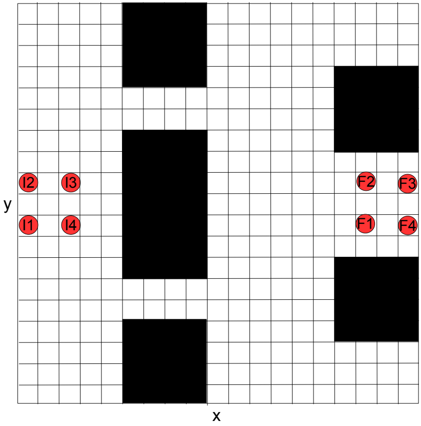


Start movie

Goal: I1 → F1, I2 → F2, I3 → F3, I4 → F4

Invariants:

- Maintain a rectangular formation
- Maintain a precedence relationship
  - The X co-ordinate of the quadrotors at I1and I2 will be always less than the X coordinate of the quadrotors at I3 and I4
- Maintain a minimum distance
  - The distance between two quadrotors is always greater than one unit

Goal

To synthesize motion plans automatically for

- a group of robots
- complex dynamics
- complex specification

Specification is given in Linear Temporal Logic (LTL)

Existing Solutions for LTL Motion Planning

- Generate a finite abstraction for the robot dynamics
- Generate a finite model for the property
- Apply a game theoretic algorithm to generate a high level plan
- Generate low level control signals that satisfy the bisimulation property

Work by Kress-Gazit, Fainekos, Pappas, Karaman, Frazzoli, Kavraki, Verdi, Topcu, Murray, Belta, Rus and others..

Computationally expensive.. Not suitable for multi-robot systems

Our Approach

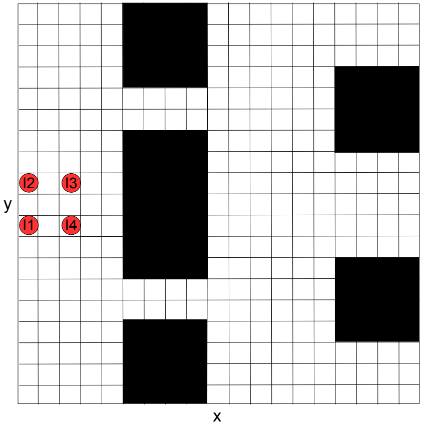
- We assume availability of a set of precomputed control laws for each robot
  - motion primitives
- We use an off-the-shelf SMT solver to generate motion plans composing these motion primitives

State of a Multi-Robot System

State of a robot i: $\phi_i = \langle q, X \rangle$

- q - Velocity configuration
- X - Position

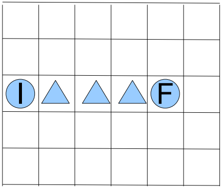
State of the multi-robot system:
$\Phi = [\phi_1, \ldots, \phi_N]$

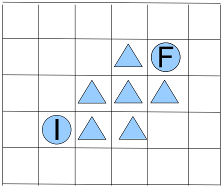


Motion Primitive

A motion primitive is formally defined as a 7-tuple: $\langle u, \tau, q_i, q_f, X_{rf}, W, cost \rangle$.

- u - a precomputed control input
- τ - the duration for which the control signal is applied
- q_i - initial velocity configuration
- q_f - final velocity configuration
- X_{rf} - relative final position
- W - the set of relative blocks through which the robot may pass
- $cost$ - an estimated energy consumption for executing the control law





Note: Motion Primitives are position oblivious

Motion Planning Problem

An input problem instance $\mathcal{P} = \langle N, I, F, PRIM, OBS, \xi \rangle$

- N - Number of robots
- I - Initial state of the group of robots
- F - Final state of the group of robots
- $PRIM = [PRIM_1, PRIM_2, \ldots, PRIM_N]$
- OBS - the set of obstacles
- ξ - $\Box\Psi$, conjunction of a set of invariant properties

Definition (Motion Planning Problem)

Given an input problem $\mathcal{P}$ and a positive integer L, synthesize a motion plan of length $L + 1$

Motion Plan

A *motion plan* of a multi-robot system for an input problem instance $\mathcal{P} = \langle N, I, F, PRIM, OBS, \Box\Psi \rangle$ is defined as a sequence of states $\Phi = (\Phi(0), \Phi(1), \ldots, \Phi(L))$ such that

- $\Phi(0) \in I$
- $\Phi(L) \in F$
- $\Phi(0) \models \Psi$

and the states are related by the transitions in the following way:

$$\Phi(0) \xrightarrow{Prim_1} \Phi(1) \xrightarrow{Prim_2} \Phi(2) \ldots \Phi(L-1) \xrightarrow{Prim_L} \Phi(L)$$

Transition Constraints

$\Phi_1 = [\phi_{11}, \ldots, \phi_{1N}]$, $\Phi_2 = [\phi_{21}, \ldots, \phi_{2N}]$

$Prim = [prim_1, \ldots, prim_N]$, where $prim_i \in PRIM_i$.

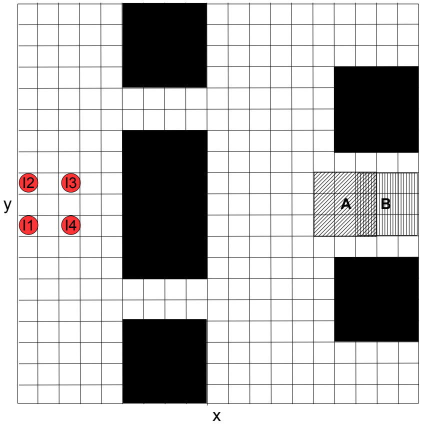
A transition

$$\Phi_1 \xrightarrow{Prim} \Phi_2$$

is associated with the following constraints:

- $\forall i \in \{1, \ldots, N\} : \phi_{1i}.q = prim_i.q_i$
- $\forall i \in \{1, \ldots, N\} : \phi_{2i}.q = prim_i.q_f$
- $\forall i \in \{1, \ldots, N\} : \phi_{2i}.X = \phi_{1i}.X + prim_i.X_{rf}$
- $obstacle\_avoidance(\Phi_1, \Phi_2, Prim, OBS)$
- $collision\_avoidance(\Phi_1, \Phi_2, Prim)$
- $(\Phi_1 \models \Psi) \rightarrow (\Phi_2 \models \Psi)$

Specification 2

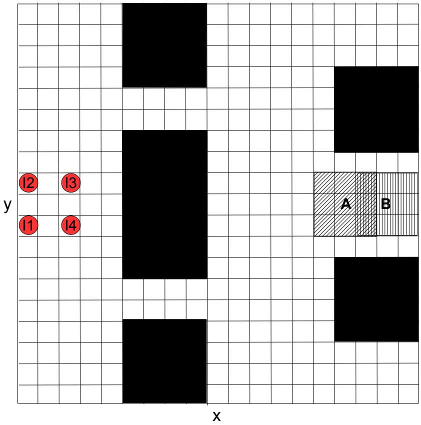


Goal: (I1 and I2) → B
(I3 and I4) → A

Invariants:

- Maintain a rectangular or linear formation
- Maintain a minimum distance
  - The distance between two quadrotors is always greater than one unit

Specification 2



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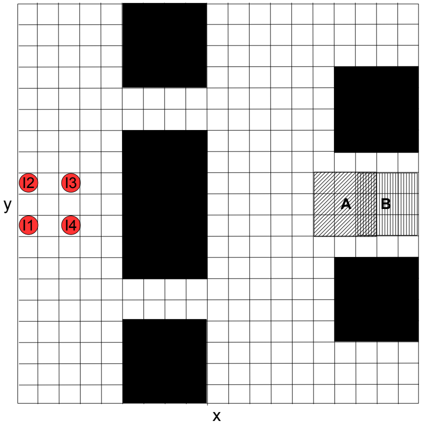
No motion plan that satisfies the formation constraint exists

Specification 2



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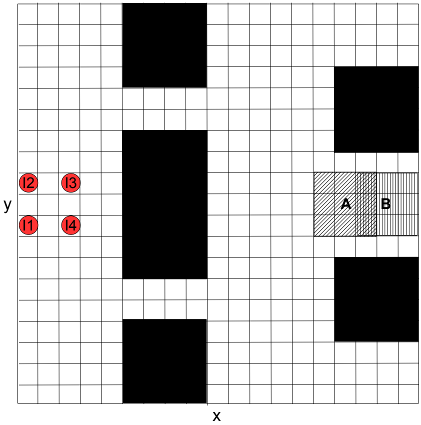
Finding Optimal Trajectory

- Find the least number of motion primitives that can generate a valid trajectory

- Among all trajectories that use the least number of motion primitives, find the one that incurs the least cost

Specification 2



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Completeness

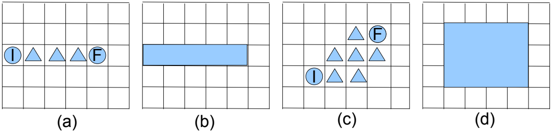
The completeness is with respect to the given set of motion primitives.

Given a positive integer L, If there exists a trajectory of length L using the given set of motion primitives, our technique is able to generate that trajectory.

Runtime

Destination Specification	Without rectangular abstraction	With rectangular abstraction
Spec 1	4m51s	2m06s
Spec 2	5m59s	3m25s

Table: Experimental results on two case studies.



Current and Future Work

- How to handle arbitrary LTL specification in our framework?

  - Persistent Surveillance
  [Belta and others, ICRA 2012, CDC 2012, ...]

- How to deal with change in environment?

  - Patching task level robot controllers
  [LivingstonPrabhakarJoseMurray, ICRA 2013]

- How to scale our framework for a large number of robots?

  - Distributed motion planning
  [TurpinMichaelKumar, ICRA 2012, JRR 2014]

Thank You!!