

CS201A: Math for CS I/Discrete Mathematics

#6

Max marks:75

Due on/before:23.00, 15-Nov-2017.

9-Nov-2017

1. (a) Two players, A , B take turns at rolling a die; they each need a different value to win. If they do not roll the required value, play continues. At each of their attempts A wins with probability α whereas B wins with probability β . What is the probability that A wins if s/he rolls first? What is it if s/he rolls second?
- (b) Two coins, A and B , show heads with respective probabilities α and β . They are tossed alternately, giving $ABABAB\dots$. Find the probability of the event E that A is first to show a head.
- (c) Suppose you toss three fair coins. At least two must be alike, and the probability that the third is a head or a tail is $\frac{1}{2}$. So the probability that all three are the same is $\frac{1}{2}$. But probability that all three are same is clearly $P(HHH) + P(TTT) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$. What is wrong with the argument?
- (d) A flips $n+1$ fair coins and B flips n fair coins. A wins if s/he has more heads than B . Find the probability that A wins.

[(5,5),5,5,5=25]

2. (a) A fair die has s sides. Find the expression for the probability p that each side has turned up at least once if it was rolled r times?
- (b) A large number of students in a lecture room are asked to state their birthday one by one. The first student who shares a birthday with someone whose birthday has been already announced wins a prize. When (that is sequence number) should you be asked to state your birthday so that you have the best chance of winning the prize.

[10,10=20]

3. Example Let X have mass function $f(x)$. Find the mass functions of the following functions of X .

- (a) $-X$
- (b) $X^+ = \max(0, X)$
- (c) $X^- = \max(0, -X)$
- (d) $|X| = X^+ + X^-$
- (e) $\text{sgn}(X) = \begin{cases} \frac{X}{|X|}, & X \neq 0 \\ 0, & X = 0 \end{cases}$

[2,3,3,3,4=15]

4. (a) Supposing a box contains n cards numbered 1 to n . You pick one card at random. It has the number X . Find $E[X]$ and $E[X^2]$. What is the second central moment?
- (b) Assuming $E[X]$ exists argue that $(E[X])^2 \leq (E[|X|])^2 \leq E[X^2]$.

[(3,4,3),5=15]