CS201A: Math for CS I/Discrete Mathematics

#5

Max marks:105
Due on/before:23.00, 29-Oct-2017. 21-Oct-2017

1. A committee of n persons is to be chosen from a group of 7 women and 4 men. Find the number of ways a committee can be formed in each case below:
   (a) $n = 5$ and the committee has 3 women and 2 men.
   (b) Committee must have equal number of men and women, $n > 0$.
   (c) The committee has $n = 4$ persons and one of them is Mr. Sharma.
   (d) The committee has $n = 4$ persons and at least 2 are women.
   (e) The committee has $n = 4$ persons, two of each gender and Mr. and Mrs Sharma cannot both be in the committee.

   $[5 \times 5 = 25]$

2. (a) Define a chromatic polynomial $P_k(G)$ of a graph G as a polynomial in k that gives the number of proper $k-$colourings of graph G. Recall that a proper colouring of a graph G is one in which no two adjacent nodes have the same colour. Find the chromatic polynomial of:
      i. K_5
      ii. C_4

   (b) Derive an expression to find the number of ways to select r objects from n types of objects with repetition. Give a complete argument.

   [(5,5),10=20]

3. (a) A non-negative integer solution to the equation $x_1 + x_2 + x_3 + x_4 = 12$ is an ordered $4-$tuple (n_1, n_2, n_3, n_4) where$n_i \geq 0$ and $x_i = n_i$. Find the number of non-negative integer solution to the above equation in each case below:
      i. $x_i \geq 0$.
      ii. $x_i \geq 1$.
      iii. $x_1 \geq 2$, $x_2 \geq 2$, $x_3 \geq 4$, $x_4 \geq 0$

   (b) You have 7 friends. Find the number of ways to invite a different subset of 3 friends for dinner on 7 successive nights such that each pair of friends are together at just one dinner.

   $[(3 \times 5),10{=}25]$

4. A partition of n is a set of positive integers that add up to n. n itself is a trivial partition of n.
   (a) Find the generating function for a_n, the number of partitions that add up to at most n.
   (b) Find a generating function for a_n, where n is partitioned into 3 parts such that no part is larger than the sum of the other two.
   (c) Find a generating function for a_n, the number of different (incongruent) triangles with integral sides whose perimeter is n.

   $[3 \times 5{=}15]$

5. Use generating functions to solve the following set of simultaneous recurrences. $a_n = a_{n-1}+b_{n-1}+c_{n-1}$, $b_n = 3^{n-1}-c_{n-1}$, $c_n = 3^{n-1} - b_{n-1}$. Initial conditions are: $a_1 = b_1 = c_1 = 1$. [20]