CS201A: Math for CS I/Discrete Mathematics
#3

Max marks:90
Due on/before:23.00, 13-Sep.-2017. 4-Sep.-2017

1. (a) Let G be a graph with 10 nodes and 28 edges. Argue that G has a cyle of length 4.
   (b) $H = (V', E')$ is said to be an **induced subgraph** of graph $G = (V, E)$ if $V' \subseteq V$ $(u, v) \in E'$ iff $(u, v) \in E$ where $u, v \in V'$.

   Let G be a graph with 10 nodes and 38 edges. Prove that G contains K_4 as an induced sub-graph.
   (c) An **automorphism** is an isomorphism between G and G. How many automorphisms do the following graphs have (assume all nodes are labelled):
      i. K_n.
      ii. C_n
      iii. P_n
      iv. S_n - Star n. This graph has one node with degree $n-1$ and other $n-1$ nodes have degree 1.
   (d) If G is a simple graph where each node has degree k then show that G has a cycle of length at least $k+1$.

   [8,8,(4x4),8=40]

2. (a) Let $G = (V, E)$ be a graph with n nodes. Also, for distinct $u, v \in V$ let $deg(u) + deg(v) \geq n$.
      i. First show that G is connected.
      ii. Then show that G has a H-cycle. (Do not use Ore's or Dirac's theorems in your proof.)
      iii. Show that if $deg(u) + deg(v) \geq n-1$ the above does not hold.
   (b) Prove Dirac's theorem without using Ore's theorem. If $G = (V, E)$ is a simple graph with n nodes and $\forall v \in V$, $deg(y \geq \frac{n}{2})$ then G has a H-cycle.

   [(4,6,5),10=25]

3. Two H-cycles in a graph are said to be different when their edge sets are different.
   (a) How many different H-cycles does K_n contain? Derive/justify your answer.
   (b) $K_{m,n}$ is a complete bi-partite graph where $|V_1| = m$ and $|V_2| = n$ and mn edges connect nodes in V_1 to nodes in V_2. Assuming $m \neq n$ find the number of H-cycles in $K_{m,n}$. Derive/justify your answer. What happens when $m = n$?

   [10,(10,5)=25]