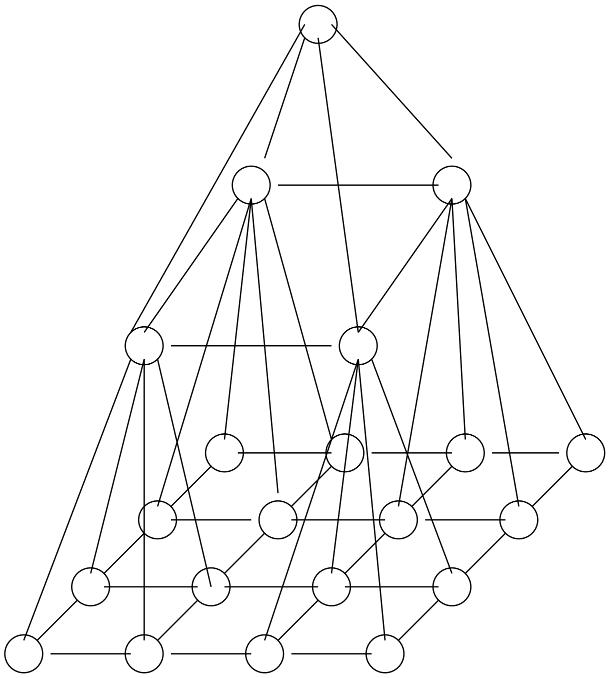
Binary Tree Network

- Processors are arranged in form of a full binary tree.
- So if the depth of tree is $k-1$, such a network will have 2^k-1 nodes.
- Every interior tree has 3 links.
- The diameter of the tree network is $2(k-1)$.
- The bisection width is 1 only, because we can just snap one link from the root to one of its children to produce two subtrees of same size.
- The link length will increase with number of nodes.

Pyramid Network

- Combines the advantages of both tree and mesh.
- Reduces the diameter compared to a mesh.
- Number of processor in a pyramid with k^2 node at the base:

$$1 + 4 + 4^2 + \ldots + k^2 = \frac{4^{\log k+1} - 1}{4 - 1} = \frac{4k^2 - 1}{3}$$



Pyramid Network

- The diameter is $2 \log k$.
- Maximum number of links per node is 9.
- But link length increases with size.
- For dividing a pyramid of size k^2 into 2 halves of equal sizes, we need to delete
  - k links at the base, $k/2$ links at level up, $k/2^2$ links at two levels up, and so on, and
  - finally, 2 links from the root.
- So the number of links to be deleted is given following expression

$$k + \frac{k}{2} + \frac{k}{2^2} + \ldots + \frac{k}{2^{\log k - 1}} + 2$$

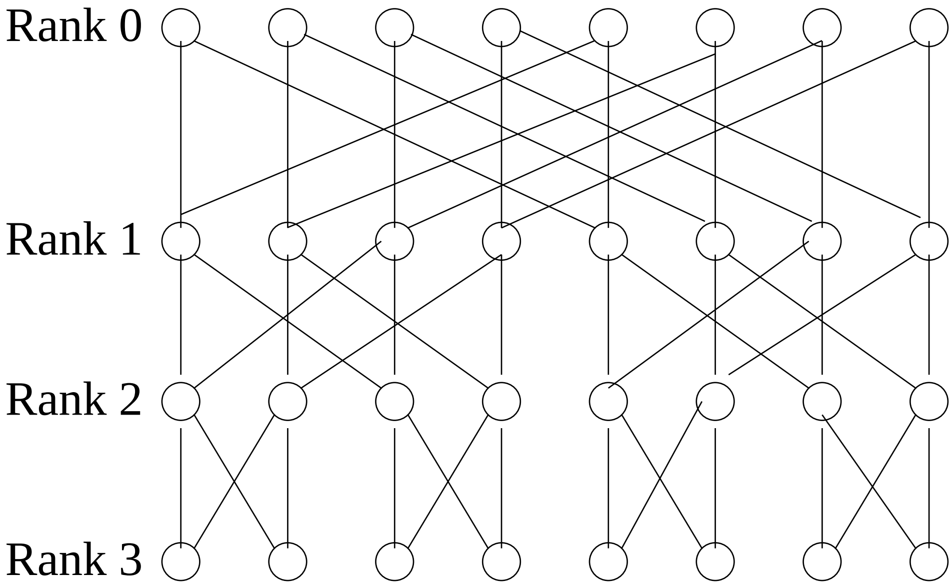
- Simplifying the above expression we get

$$k \frac{1 - (\frac{1}{2})^{\log k}}{\frac{1}{2}} + 2 = 2k - 2 + 2 = 2k$$

- So, bisection width is $2k$.

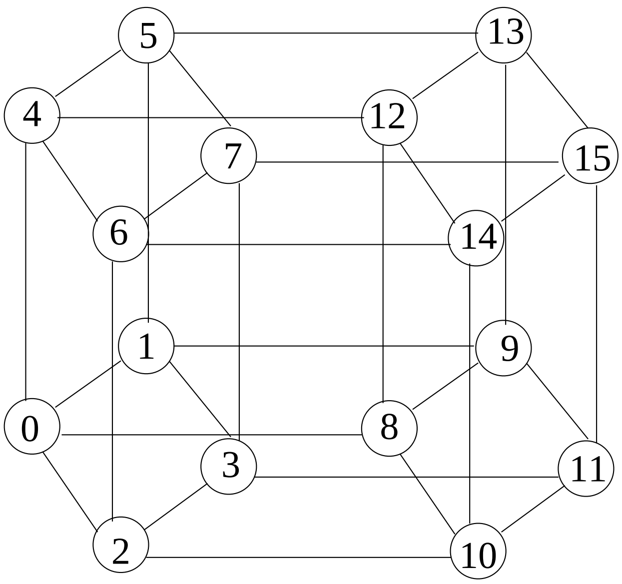
Butterfly Network

- Links between the processors that do not belong to the same row and the same column appear like wings of a butterfly.
- Processors arranged in $(k+1) \times 2^k$ array.
- Node P_{ij} is connected to 2 nodes of row $i-1$, $P_{i-1,j}$ and $P_{i-1,j^{(i)}}$



Hypercube Network

- Consists of 2^k nodes. Can be considered as butterfly with each column collapsed into a single node.
- The corresponding hypercube of a butterfly with $(k+1)2^k$ nodes will consists of 2^k nodes.
- The links between the nodes in the hypercube are determined by the links between columns of the butterfly.
- So processor with index i is connected to processor with index $i^{(b)}$ for all $0 \leq b \leq k-1$.



Hypercube Network

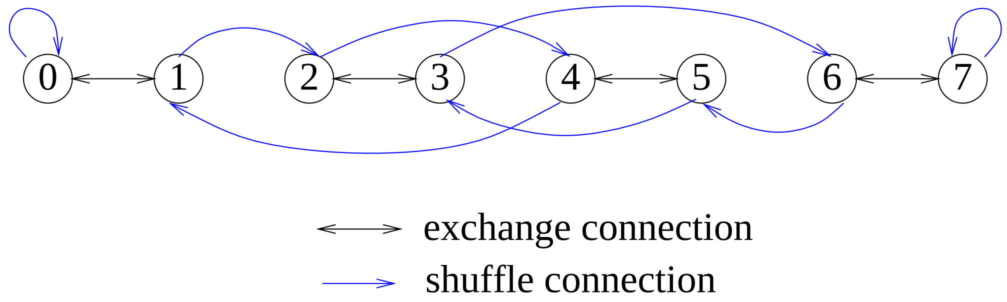
- The diameter of a 2^k hypercube is k.
- The bisection width is 2^{k-1} as all the links of highest dimension must be removed to produce two hypecubes of half the size and one reduced dimension.
- The number of links per node is dependent on the number of dimension which is k or logarithmic in size of the hypercube.
- The length of the longest link is a function of size of the hypercube.

Hypercube Network

- We can extend the binary hypercube network to generalized cube network.
- In generalized cube network the processor indices are represented by a number of base b digits.
- Assume i is represented as $x_0x_1 \ldots x_{k-1}$ base b number.
- Then i is connected to $x_0x_1 \ldots \overline{x_j} \ldots x_{k-1}$, for $0 \leq j \leq k-1$ and $\overline{x_j}$ represents the a base b digit other than x_j.
- For example if $b = 3$ and $k = 3$ then the processor indexed as 121 will be connected to 6 processors: 021, 221, 111, 101, 120, 122.
- Unless a generalized hypercube network of base b has b^k processors it will be called an incomplete generalized hypercube.
- A generalized hypercube of N nodes with base b is represented as (N, b, k), where $N \leq b^k$ is the number of nodes in the network.

Shuffle Exchange Network

- Two kind of connection per node: **shuffle** and **exchange**.
- For shuffle connection node i is connected to $2i \bmod (n-1)$, where $n = 2^k$.
- Exchange connection is between i and $i^{(b)}$.
- A n/w of 8 nodes is shown below.



- The perfect shuffle idea comes from shuffling cards.
- Let the cards be numbered as 0, 1, 2, . . . , 7.

Shuffle Exchange Network

- Divide these into two decks: 0, 1, 2, 3 and 4, 5, 6, 7.
- Now if we shuffle these two decks the cards will be arranged in the sequence 0, 4, 1, 5, 2, 6, 3, 7.
- So final position of card indexed i is determined by shuffle link from node i.
- Let $i_{k-1}i_{k-2}\ldots i_1 i_0$ be the address of of node i expressed in binary then by a shuffle operation the data at i will be transferred to node at $i_{k-2}i_{k-3}\ldots i_1 i_0 i_{k-1}$.
- So shuffle is equivalent to a left cyclic rotation of address by 1 bit.
- If we apply k shuffle operation the data comes back to the orginal node.

Shuffle Exchange Network

- Every node has exactly two outgoing links and two incoming links.
- Length of longest link is a function of size.
- A network with 2^k nodes has diameter $2k-1$.
- The bisection width is at least $2^{k-1}/k$.

Summary

- We studied the requirement of processing power for the grand challenge problems.
- The processing power is limited by the amount of information that can move from one side of a square chip area to the other.
- This limitation is expressed succintly by the so called AT^2 law.
- Reducing time by a factor will increase the area by the square of that factor.
- So, not only building faster individual components is difficult, it is cheaper to use many existing slow components and build multicomponent systems to derive desired processing power.
- So, parallel computing assumed significance and many parallel computers were built.

Summary

- We considered architecture of some parallel models on the basis of parallelism in data and instruction streams.
- Classification schemes: Flynn and Handler introduced.
- Then we talked about PRAM and its many variations: EREW, CREW, CRCW.
- Paradigms: Balanced binary tree, prefix sums, list ranking partitioning discussed.
- Optimal merging was considered as a example of partitioning technique.
- Processor organization of distributed memory machine are discussed: Mesh, Tree, Pyramid, Hypercube, and Shuffle-exchange.

