Polygon

- A polygon is a piece-wise linear closed curve.
- The point joining two adjacent sides of polygon is called a vertex.
- It is simple if polygon does not cross itself.
- Set of points in the plane enclosed by the polygon is called its interior.
- Set of points in the plane sorrounding the polygon is called its exterior.
- Set of points on the polygon is called its boundary.
- If the line joining any points either within the interior or on the boundary of a polygon lies entirely inside/on the polygon, then the polygon is convex.

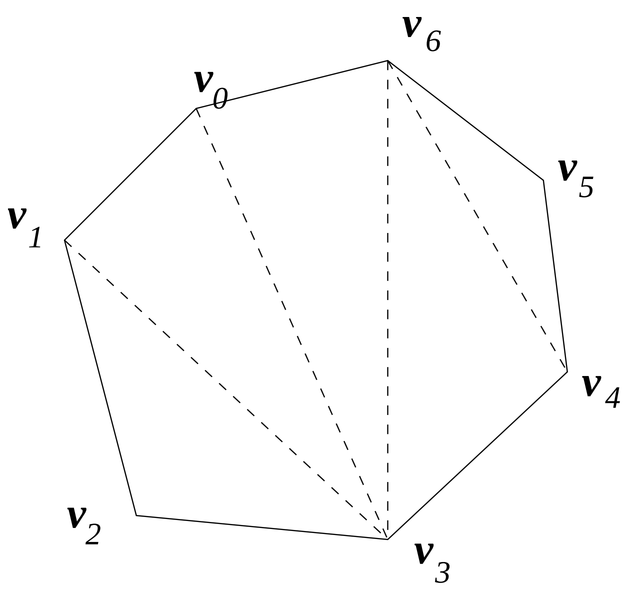
Convex Polygon

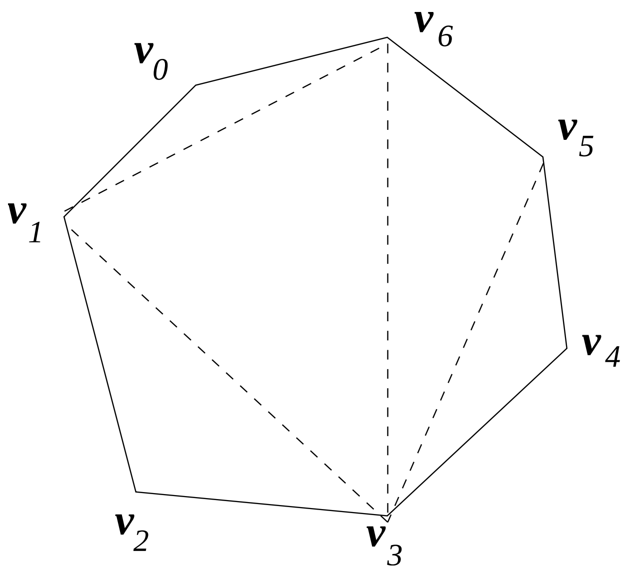
- Consider a convex polygon.
- We can list the sides of such a polygon in counter-clockwise order.
- $P = \langle v_0, v_1, \ldots, v_{n-1} \rangle$.
- Two adjacent points are $v_i, v_{(i+1) \mod n}$.
- Two non-adjacent points when joined gives rise to a chord.
- A chord $\overline{v_i v_j}$ divides the polygon into two polygons.

$$\langle v_i, v_{i+1}, \ldots, v_j \rangle \text{ and } \langle v_j, v_{j+1}, \ldots, v_i \rangle$$

- Triangulation is a set of chords that partitions a polygon into disjoint triangles.
- No two chords should intersect.

Triangulation



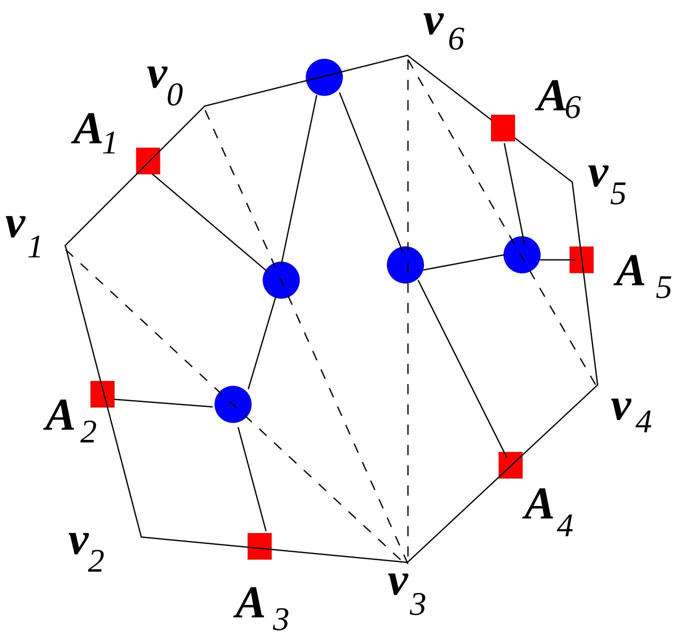


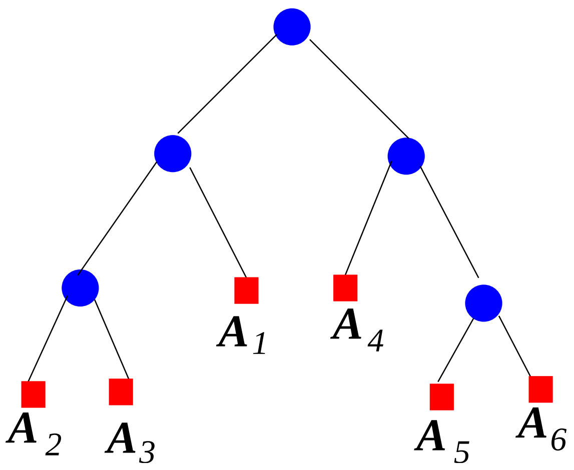
Optimal Triangulation

- Suppose there is weight function associated with the triangles formed by joining chords of P.
- The problem is to find the set of chords that generate minimum total weight of triangles.
- A simple weight function may be

$$w(\triangle v_i v_j v_k) = |v_i v_j| + |v_j v_k| + |v_k v_i|$$

Parenthesizing Problem Revisited





Optimal Triangulation

- We formed the tree with
  - side $\overline{v_0 v_6}$ as root.
  - each chord is an internal node.
  - other remaining sides as leaf nodes.
- The root $\overline{v_0 v_6}$ is one side of the triangle $\triangle v_0, v_6 v_3$ and the chords $\overline{v_1 v_3}$ and $\overline{v_6 v_3}$ are other two sides.

Optimal Triangulation

- $\triangle v_0 v_6 v_3$ divides the polygon into three parts
  - $\triangle v_0 v_6 v_3$
  - polygon $P' = \langle v_0, v_3, v_2, v_1 \rangle$
  - polygon $P'' = \langle v_3, v_4, v_5, v_6 \rangle$
- The triangle represent the internal node in the tree representation of the original polygon with the two polygons P' and P'' as its left and right children.

Optimal Triangulation

- We can recursively partition the two polygons P' and P''.
- So an n sided polygon can be represented by a tree with $n-1$ leaves by triangulation.
- Inversely given a tree triangulation we can produce a tree representation.
- In fact matrix chain product is a special case of triangulation problem.
- Let matrix A_i be of dimension $p_{i-1} \times p_i$.
- We define the weight function for triangulation as
$$w(\triangle v_i v_j v_k) = p_i p_j p_k$$
- An optimal triangulation with respect to the above weight function gives an optimal parenthesization of the matrices.

Optimal Triangulation

```
matrixChainOrder(p) {
    n = lenght[p] − 1;
    for (i = 1; i ≤ n; i++) m[i, i] = 0;
    for (l = 2; l ≤ n; l++)
        for (i = 1; i <= n − l + 1; i++) {
            j = i + l − 1;
            m[i, j] = i + l − 1;
            for (k = i; k ≤ j − 1; k++) {
                q = m[i, k] + m[k + 1, j]
                    +w(△v_i v_k v_j);
                if (q < m[i, j]) {
                    m[i, j] = q; s[i, j] = k;
                }
            }
        }
    return m, s
}
```

Optimal Substructure

- Consider polygon of $P = \langle v_0, v_1, \ldots, v_n \rangle$ with $n+1$ vertex.
- Suppose the chords $\overline{v_k v_0}$ and $\overline{v_k v_n}$ belong to an optimal triangulation $\mathbf{T}$ of P.
- Consider the partitions P due to $\mathbf{T}$ into $\triangle v_0 v_k v_n$, two polygons $P_1 = \langle v_0, v_1, \ldots, v_k \rangle$, and $P_2 = \langle v_k, v_{k+1}, \ldots, v_n \rangle$.
- The triangulation of P_1 and P_2 as determined by $\mathbf{T}$ must also be optimal.
- Otherwise a lesser weight triangulation of either P_1 or P_2 will determine a lesser weight triangulation for P.

