## LINEAR DIOPHANTINE EQUATION

An equation of the form

$$ax + by = c$$

where $a, b, c$ are integers and $a, b$ not both zero is called a linear Diophantine equation.

- We seek to find a pair of integers $x_0, y_0$ such that
$$ax_0 + by_0 = c.$$
- The equation $3x + 6y = 18$ has a number of solutions:
$$3.4 + 6.1 = 18$$
$$3(-6) + 6.6 = 18$$
$$3.10 + 6(-2) = 18$$
- Whereas the equation $2x + 10y = 17$ has no solution (obvious since the LHS is an even integer whereas the RHS gives an odd integer)

- **Condition for Solvability:** The Linear Diophantine Equation $ax + by = c$,has a solution if and only if $d$ divides $c$,where $d = gcd(a, b)$.If $x_0, y_0$ is any particular solution of this equation,then all other solutions are given by

$$x = x_0 + (b/d)t, y = y_0 - (a/d)t$$

for varying integers $t$.

- Example::Consider the Linear Diophantine Equation:

$$172x + 20y = 1000$$

- Calculate gcd(172,20) using Euclid's algorithm

$$172 = 8.20 + 12$$

$$20 = 1.12 + 8$$

$$12 = 1.8 + 4$$

$$8 = 2.4$$

- Hence,gcd(172,20)=4

- Since 4 divides 1000 a solution to this equation exists
- Obtain 4 as a linear combination of 172 and 20 by working backwards

$$4 = 12 - 8$$
$$= 12 - (20 - 12)$$
$$= 2.12 - 20$$
$$= 2(172 - 8.20) - 20$$
$$= 2.172 + (-17)20$$

- Multiplying by 250

$$1000 = 250.4 = 250[2.172 + (-17)20]$$
$$= 500.172 + (-4250)20$$

- Hence,one solution is given by $x = 500$ and $y = -4250$
- All other solutions are given by

$$x = 500 + (20/4)t = 500 + 5t$$
$$y = -4250 - (172/4)t = -4250 - 43t$$