# Topics Covered in Lecture 33 & 34

31-October-2002

- Home work #6, #7 and #8 solutions are discussed.
- **MARKOV CHAINS WITH ABSORBING STATES**
  **Example 8.34**

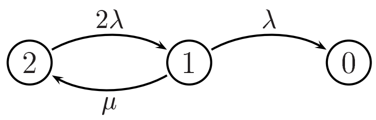


**Figure 8.58**. The state diagram for Example 8.34

$$
\begin{aligned}
\frac{d\pi_2(t)}{dt} &= -2\lambda\pi_2(t) + \mu\pi_1(t), \\
\frac{d\pi_1(t)}{dt} &= 2\lambda\pi_2(t) - (\lambda+\mu)\pi_1(t), \\
\frac{d\pi_0(t)}{dt} &= \lambda\pi_1(t).
\end{aligned}
$$

Using the technique of Laplace transform, we can reduce this system to

$$
\begin{aligned}
s\bar{\pi}_2(s) - 1 &= -2\lambda\bar{\pi}_2(s) + \mu\bar{\pi}_1(s), \\
s\bar{\pi}_1(s) &= 2\lambda\bar{\pi}_2(s) - (\lambda+\mu)\bar{\pi}_1(s), \\
s\bar{\pi}_0(s) &= \lambda\bar{\pi}_1(s).
\end{aligned}
$$

$$\bar{\pi}_0(s) = \frac{2\lambda^2}{s[s^2 + (3\lambda+\mu)s + 2\lambda^2]}.$$

$$R(t) = 1 - \pi_0(t).$$

$$f_Y(t) = \frac{d\pi_0(t)}{dt}$$

$$L_Y(s) = \frac{2\lambda^2}{\alpha_1 - \alpha_2}\left(\frac{1}{s+\alpha_2} - \frac{1}{s+\alpha_1}\right)$$

$$f_Y(t) = \frac{2\lambda^2}{\alpha_1 - \alpha_2}(e^{-\alpha_2 t} - e^{\alpha_1 t})$$

$$R(t) = \frac{2\lambda^2}{\alpha_1 - \alpha_2}\left(\frac{e^{-\alpha_2 t}}{\alpha_2} - \frac{e^{-\alpha_1 t}}{\alpha_1}\right).$$

$$E[Y] = \frac{3}{2\lambda} + \frac{\mu}{2\lambda^2}$$

- **Example 8.35**

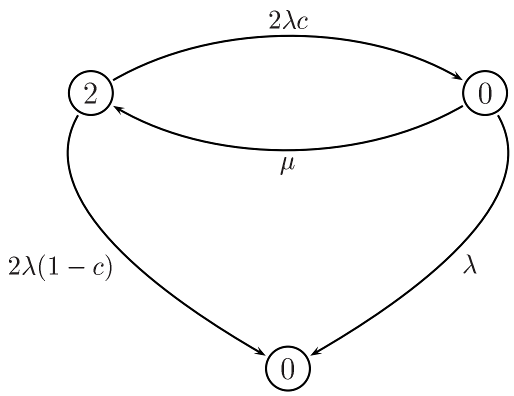


**Figure 8.59**. The state diagram of a duplex system with imperfect coverage

$$E[X] = \frac{\lambda(1+2c)+\mu}{2\lambda[\lambda+\mu(1-c)]}$$

- **Example 8.38**

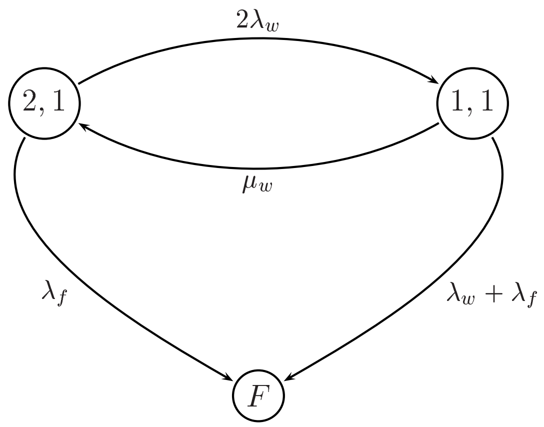


**Figure 8.64**. CTMC reliability models for the WFS example with repair