Topics Covered in Lecture 27

5-October-2002

- **The Pure Birth Process** If the death rates $\mu_k = 0$ for all $k = 1, 2, 3, \ldots$, we have a **pure birth process**.

$$\begin{aligned} \frac{d\pi_0(t)}{dt} &= -\lambda\pi_0(t) & k = 0 \\ \frac{d\pi_k(t)}{dt} &= \lambda\pi_{k-1}(t) - \lambda\pi_k(t) & k \geq 1 \end{aligned}$$

$$\pi_0(0) = 1, \pi_k(0) = 0, \text{ for } k \geq 1$$

$$\bar{\pi}_k(s) = \int_0^\infty e^{-st}\pi_k(t)dt$$

$$\bar{\pi}_0(s) = \frac{1}{s+\lambda}$$

$$\bar{\pi}_k(s) = \frac{\lambda}{s+\lambda}\bar{\pi}_{k-1}(s)$$

- **Non Homogeneous Poisson Process** Here the arrival rate is $\lambda(t)$. Here the t used is global clock. If t is a local clock then it becomes a "semi-markov chain".

$$\pi_k(t) = \frac{[m(t)]^k}{k!}e^{-m(t)}, \qquad k \geq 0$$

$$m(t) = \int_0^t \lambda(x)dx.$$

- **Example 8.14 (Software Reliability Growth Models)**
  The failure intensity is given by

$$\lambda(t) = abe^{-bt}$$

The expected number of software failures by time t is

$$m(t) = E[N(t)] = a(1 - e^{-bt})$$

The instantaneous failure intensity is given by

$$\lambda(t) = b[a - m(t)]$$

- **Pure Death Processes**
- **Death Process with a Linear Rate**

$$\pi_k(t) = \binom{n}{k} (e^{-\mu t})^k (1 - e^{-\mu t})^{n-k}, \qquad 0 \le k \le n, t \ge 0.$$

- **NON-BIRTH-DEATH PROCESSES**
- **Availability Models**
- Example 8.20 (A Preventive Maintanance Model) is discussed.