Topics Covered in Lecture 25

3-October-2002

- The LST of the completion time distributions are

$$L_{prs}(s) = exp\left\{-\frac{s^2+(\lambda+\mu)s}{s+\mu}x\right\} \qquad \text{Preemptive resume case}$$

$$L_{prt}(s) = \frac{e^{-(s+\lambda)x}}{1-\frac{\lambda}{s+\lambda}\frac{\mu}{s+\mu}(1-e^{-(s+\lambda)x})} \qquad \text{Preemptive repeat case}$$

- **Example 8.8 (Cyclic Queuing Model of a Multiprogramming System)**
  The steadystate probabilities are given by

$$\pi_i = \left(\frac{\lambda}{\mu q_1}\right)^i \pi_0 = \rho^i \pi_0, \text{ and } \pi_0 = \frac{1}{\sum\limits_{i=0}^{n} \rho^i}$$

$$\pi_0 = \begin{cases} \frac{1-\rho}{1-\rho^n}, & \rho \neq 1, \\ \frac{1}{n+1}, & \rho = 1. \end{cases}$$

  The CPU utilization is give by

$$U_0 = 1 - \pi_0 = \begin{cases} \dfrac{\rho-\rho^{n+1}}{1-\rho^{n+1}}, & \rho \neq 1 \\ \dfrac{n}{n+1}, & \rho = 1 \end{cases}$$

  The throughput is given by

$$E[T] = \mu q_0 U_0.$$

$$\rho = \frac{\lambda}{\mu q_1} = \frac{E[B_0]}{E[B_1]}.$$

$$U_0 = \frac{\rho}{1-\rho} = \frac{1}{E[B_0]+E[B_1]}.$$

- **Machine Repairman Model**

  The steady state probabilities are given here as

$$\pi_k = \pi_0 \prod_{i=0}^{k-1} \frac{\lambda(M-i)}{\mu}, 0 \le k \le M,$$

or

$$\pi_k = \pi_0 \left(\frac{\lambda}{\mu}\right)^k \frac{M!}{(M-k)!} = \pi_0 \rho^k \frac{M!}{(M-k)!}.$$

Hence

$$\pi_0 = \frac{1}{\sum_{k=0}^{M} \rho^k \frac{M!}{(M-k)!}}$$