Topics Covered in Lecture 24

1-October-2002

- **The $M/M/m$ Queue** Take a queuing system with arrival rate λ , having $m \geq 1$ servers, each with a service rate μ, share a common queue. This gives rise to a birth-death model with the following rates:

$$\begin{aligned} \lambda_k &= \lambda, \qquad k = 0, 1, 2, \ldots \\ \mu_k &= \begin{cases} k\mu, & 0 < k < m, \\ m\mu, & k \geq m \end{cases} \end{aligned}$$

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$$\sum_{k=0}^{\infty} \pi_k = 1$$

$$\sum_{k=0}^{m-1} \pi_k + \sum_{k=m}^{\infty} \pi_k = 1.$$

$$\frac{\lambda}{m\mu} < 1 \Rightarrow \lambda < m\mu.$$

- # of jobs in the system at an instance of time is

$$E[M] = m\rho = \frac{\lambda}{\mu}$$

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$$\begin{aligned} p(\text{queuing}) &= \sum_{k=m}^{\infty} \pi_k = \frac{\pi_m}{1-\rho} \\ &= \frac{(m\rho)^m}{m!} . \frac{\pi_0}{1-\rho}. \end{aligned}$$

- **Finite State Space**

$$\pi_i = \rho^i \pi_0, \qquad 0 \le i \le n,$$

$$\pi_0 = \frac{1}{\sum_{i=0}^{n} \rho^i} = \begin{cases} \dfrac{1-\rho}{1-\rho^{n+1}}, & \rho \neq 1 \\ \dfrac{1}{n+1}, & \rho = 1 \end{cases}$$

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Measure	Rewardrate assignment	expected steady-state reward rate
Mean number in system	$r_j = j$	$\dfrac{\rho}{\rho-1} - \dfrac{n+1}{1-\rho^{n+1}}\rho^{n+1}$
Loss probability	$r_n = 1, r_j = 0 (j \neq n)$	$\pi_n = \dfrac{1-\rho}{1-\rho^{n+1}\rho^n}$
Throughput	$r_j = \mu (j \neq 0), r_0 = 0$ or $r_j = \lambda (j \neq n), r_n = 0$	$\mu(1-\pi_0) = \lambda(1-\pi_n)$