Topics Covered in Lecture 22

13-September-2002

- $$\pi_0(t+h) \quad = \quad \pi_0(t)[1-\lambda h+o(h)]+\pi_1(t)[\mu h+o(h)]$$

- Kolmogorov differential equation.

$$\frac{d\pi_0}{dt} = -\lambda\pi_0(t)+\mu\pi_1(t)$$

$$\frac{d\pi_k}{dt} = -(\lambda+\mu)\pi_k(t)+\lambda\pi_{k-1}(t)+\mu\pi_{k+1}(t).$$

- in steady state flow in = flow out. The equations become

$$\lambda\pi_0 = \mu\pi_1$$

$$(\lambda+\mu)\pi_k = \lambda\pi_{k-1}+\mu\pi_{k+1}, k = 1,2,\ldots$$

$$\pi Q = 0$$

$$\begin{aligned}\pi_k &= \left(\tfrac{\lambda}{\mu}\right)\pi_{k-1} \\ &= \left(\tfrac{\lambda}{\mu}\right)^k\pi_0\end{aligned}$$

$$\sum_{k=0}^{\infty}\pi_k = 1$$

$$\pi_0\left(\sum_0^{\infty}\left(\frac{\lambda}{\mu}\right)^k\right) = 1$$

$$\pi_0 = 1-\frac{\lambda}{\mu}$$

$$\pi_k = \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^k$$

- The $M/M/1$ Queue

$$\# \text{ of jobs in the system} = \frac{\rho}{1-\rho} \text{ Here } \rho = \text{traffic intensity}$$

- Little's result

$$E[R] = \lambda E[N]$$

$$\begin{aligned} E[R] &= \sum_{k=0}^{\infty} \frac{k}{\lambda} \pi_k. \\ &= \frac{1/\mu}{1-\rho} \end{aligned}$$