Topics Covered in Lecture 19

11-September-2002

- Chapter 6 named **Stochastic Processes** is started.

- **Definition (Stochastic Process).** A stochastic process is a family of random variables $\{X(t)|t \in T\}$, defined on a given probability space, indexed by the parameter t, where t varies over an index set T.

- If the state space of a stochastic process is discrete, then it is called a **discrete-state process**, often referred to as a "**chain**". In this case, the state space is often assumed to be $\{0, 1, 2, \ldots\}$. Alternatively, if the state space is continuous, then we have a **continuous-state process**.

- If the index set T is discrete, then we have a **discrete-time (parameter) process**; otherwise we have a **continuous-time (parameter) process**. A discrete-time process is also called a **stochastic sequence** and is denoted by $\{X_n|n \in T\}$.

- We use the notation $F_Y/F_S/m$ to describe the queuing system. Here F_Y denotes the interarrival time distribution, F_S denotes the service time distribution and m denotes the number of servers. We use the following symbols.

M	(for memoryless) for the exponential distribution
D	for a deterministic or constant interarrival or service time
E_k	for a k-stage Erlang distribution
H_k	for a k-stage hyperexponential distribution
G	for a general distribution
GI	for general independent interarrival times

- **Definition (Renewal Process).** A **renewal process** is defined as a discrete-time independent process $\{X_n|n = 1, 2, \ldots\}$ where $X_1, X_2, \ldots,$ are independent, identically distributed, nonnegative random variables.

- **Definition (Markov Process).** A **stochastic process** $\{X(t)|t \in T\}$ is called a *Markov process* if for any $t_0 < t_1 < t_2 < \ldots < t_n < t$, the conditional distribution of *X(t)* for given values of $X(t_0), X(t_1), \ldots X(t_n)$ depends only on $X(t_n)$:

$$P[X(t) \leq x | X(t_n) = x_n, X(t_{n-1}) = x_{n-1}, \ldots, X(t_0) = x_0]$$

$$= P[X(t) \leq x | X(t_n) = x_n].$$

- If the conditional distributin function has the property of invariance with respect to the time orgin t_n:

$$P[X(t) \leq x | X(t_n) = x_n] = P[X(t - t_n) \leq x | X(0) = x_n]$$

then the Markov chain is said to be **(time-) homogeneous**.

- The Bernouli Process