Topics Covered in Lecture 17

7-September-2002

- **Example 5.3** is discussed.
- **Mixture Distributions:** The definition of conditional density (and conditional pmf) can be naturally extended to the case where X is a discrete random variable and Y is a continuous random variable(or vice versa).
- **Example 5.6** is discussed.
- **Example 5.8** is discussed.
- **Conditional Expectation:** If X and Y are continuous random variables, then the conditional density $f_{Y|X}$ is given by formula (5.8). Since $f_{Y|X}$ is a density of a continuous random variable, we can talk about its various moments. Its mean (if it exists) is called the **conditional expectation** of Y given $[X = x]$ and will be denoted by $E[Y|X = x]$ or $E[Y|x]$. Thus

$$\begin{aligned} E[Y|x] &= \int_{-\infty}^{\infty} yf(y|x)dy \\ &= \frac{\int_{-\infty}^{\infty} yf(x,y)dy}{f_X(x)}, 0 < f_X(x) < \infty. \end{aligned}$$

- In case the random variables X and Y are discrete, the conditional expectation $E[Y|x]$ is defined as

$$\begin{aligned} E[Y|X = x] &= \sum_y yP(Y = y|X = x) \\ &= \sum_y yp_{Y|X}(y|x). \end{aligned}$$

- Conditional expectation of a function $\phi(Y)$

$$E[\phi(Y)|X = x] = \begin{cases} \int_{-\infty}^{\infty} \phi(y)f_{Y|X}(y|x)dy, & \text{if } Y \text{ is continuous,} \\ \\ \sum_i \phi(y_i)p_{Y|X}(y_i|x), & \text{if } Y \text{ is discrete.} \end{cases}$$

- **Theorom of total expectation**

$$E[Y] = \begin{cases} \sum_x E[Y|X=x]p_X(x), & \text{if } X \text{ is discrete,} \\ \\ \int_{-\infty}^{\infty} E[Y|X=x]f_X(x)dx, & \text{if } X \text{ is continuous.} \end{cases}$$