Topics Covered in Lecture 16

05-September, 2002

- Markov's inequality

$$P(X \geq t) \leq \frac{\mu}{t}. \qquad t > 0$$

- Chebyshev inequality

$$P(|X - \mu| \geq t) \leq \frac{\sigma^2}{t^2}, \qquad t > 0$$

- Chapter 5 titled **Conditional Distribution and Expectation** is started.

- *Definition (Conditional pmf).* Let X and Y be discrete random variables having a joint pmf *p(x, y)*. The conditional pmf of Y given X is

$$\begin{aligned} p_{Y|X}(y|x) &= P(Y = y|X = x) \\ &= \frac{P(Y = y, X = x)}{P(X = x)} \\ &= \frac{p(x, y)}{p_X(x)} \end{aligned}$$

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$$p(x, y) = p_{Y|X}(y|x)p_X(x)$$

$$p_Y(y) = \sum_{\text{all } x} p(x, y) = \sum_{\text{all } x} p_{Y|X}(y|x)p_X(x).$$

$$\begin{aligned} p_Y(k) &= \textstyle\sum_{n=k}^{\infty} \begin{pmatrix} n \\ k \end{pmatrix} p^k(1-p)^{n-k} \frac{(\lambda t)^n e^{-\lambda t}}{n!} \\ &= e^{-\lambda t p} \frac{(\lambda t p)^k}{k!} \end{aligned}$$

- Example 5.2 is discussed.

- *Definition (Conditional pdf).* Let X and Y be continuous random variables with joint pdf *f(x, y)*. The conditional density of $f_{Y|X}$ is defined by

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}, \qquad \text{if } 0 < f_X(x) < \infty.$$

$$f(x,y) = f_X(x) f_{Y|X}(y|x) = f_Y(y) f_{X|Y}(x|y).$$

$$\begin{aligned} f_Y(y) &= \int_{-\infty}^{\infty} f(x,y)dx \\ &= \int_{-\infty}^{\infty} f_X(x) f_{Y|X}(y|x)dx. \end{aligned}$$

- Example 5.4 is discussed