Topics Covered in Lecture 15

05-September, 2002

- Moments and Transforms of some distributions.
  - Descrete Uniform Distribution

$$p_X(i) = \frac{1}{n}, \qquad 1 \le i \le n$$

$$E[X^k] = \sum_{i=1}^{n} \frac{i^k}{n}$$

$$E[X] = \frac{n+1}{2}$$

$$Var[X] = \frac{n^2-1}{12}$$

$$C_X = \sqrt{\frac{1}{3}\left(1 - \frac{2}{n+1}\right)}$$

$$G_X(z) = \frac{1}{n}\sum_{i=1}^{n} z^i$$

  - Bernouli pmf

$$p_X(0) = q, \qquad p_X(1) = p, \qquad p+q = 1.$$

$$E[X^k] = 0^k.q + 1^k.p = p, \qquad k = 1, 2, \ldots$$

$$E[X] = p.$$

$$Var[X] = pq$$

$$C_X = \sqrt{\frac{q}{p}}$$

$$G_X(z) = (1-p) + pz = 1 + pz.$$

– Binomial distribution

$$X = \sum_{i=1} nX_i$$

$$E[X] = \sum_{i=1}^{n} E[X_i] = np.$$

$$Var[X] = \sum_{i=1}^{n} Var[X_i] = npq.$$

$$C_X = \sqrt{\frac{q}{np}}$$

$$G_X(z) = (q + pz)^n \,.$$

– Geometric Distribution

$$p_X(i) = pq^{i-1}, \qquad i = 1, 2, \ldots$$

$$E[X] = \frac{1}{p}$$

$$G_X(z) = \frac{pz}{1 - qz}$$

$$Var[X] = \frac{q}{p^2}$$

$$C_X = \sqrt{1 - p}$$

– Poisson pmf

$$p_X(i) = \frac{\alpha^i e^{-\alpha}}{i!}, \qquad 0 \leq i < \infty, \qquad \alpha > 0.$$

$$E[X] = \alpha$$

$$C_X = \frac{1}{\sqrt{\alpha}}$$

$$G_X(z) = e^{-\alpha(1-z)}$$

– Continuous Uniform Distribution

$$f_X(x) = \frac{1}{b - a}, \qquad a < x < b.$$

$$E[X] = \frac{b + a}{2}$$

$$E[X^k] = \frac{b^{k+1} - a^{k+1}}{(k+1)(b-a)}.$$
$$Var[X] = \frac{(b-a)^2}{12}$$
$$C_X = \frac{b-a}{b+a}\sqrt{\frac{1}{3}}$$
$$L_X(s) = \frac{e^{-as} - e^{-bs}}{s(b-a)}.$$

– Exponential Distribution

$$f_X(x) = \lambda e^{-\lambda x}, \qquad x > 0, \qquad \lambda > 0,$$
$$E[X] = \frac{1}{\lambda},$$
$$Var[X] = \frac{1}{\lambda^2},$$
$$C_X = 1,$$
$$L_X(s) = \frac{\lambda}{\lambda + s}.$$

– Gamma Distribution

$$f_X(x) = \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}, \qquad x > 0.$$
$$E[X] = \frac{\alpha}{\lambda}.$$
$$Var[X] = \frac{\alpha}{\lambda^2},$$
$$C_X = \frac{1}{\sqrt{\alpha}}.$$
$$L_X(s) = \frac{\lambda^\alpha}{(\lambda + s)^\alpha}$$

– Hypoexponential Distribution

$$E[X] = \sum_{i=1}^{n} \frac{1}{\lambda_i}$$
$$Var[X] = \sum_{i=1}^{n} \frac{1}{\lambda_i^2}$$

$$C_X = \sqrt{\frac{\sum_{i=1}^{n} \frac{1}{\lambda_i^2}}{(\sum_{i=1}^{n} \frac{1}{\lambda_i})^2}}$$

$$L_X(s) = \prod_{i=1}^{n} \frac{\lambda_i}{\lambda_i + s}$$

– Hyperexponential Distribution

$$f(x) = \sum_{i=1}^{n} \alpha_i \lambda_i e^{-\lambda_i x}, \qquad \sum_{i=1}^{n} \alpha_i = 1, \qquad \alpha_i \geq 0, \qquad x > 0.$$

$$E[X] = \sum_{i=1}^{n} \frac{\alpha_i}{\lambda_i}.$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$C_X^2 = \frac{Var[X]}{(E[X])^2}$$

$$L_X(s) = \sum_{i=1}^{n} \frac{\alpha_i \lambda_i}{\lambda_i + s}.$$

– Weibull Distribution

$$f(x) = \lambda \alpha x^{\alpha-1} e^{-\lambda x^\alpha}, \qquad \lambda > 0, \alpha > 0, x > 0.$$

$$E[X] = \int_0^\infty \lambda x \alpha x^{\alpha-1} e^{-\lambda x^\alpha}\, dx.$$

$$E[X] = \left(\frac{1}{\lambda}\right)^{1/\alpha} \Gamma\left(1 + \frac{1}{\alpha}\right)$$

$$Var[X] = \left(\frac{1}{\lambda}\right)^{2/\alpha} \left[\Gamma\left(1 + \frac{2}{\alpha}\right) - \left[\Gamma\left(1 + \frac{1}{\alpha}\right)\right]^2\right]$$

$$C_X = \sqrt{\frac{\Gamma(1 + 2/\alpha)}{[\Gamma(1 + 1/\alpha)]^2} - 1}$$

– Pareto Distribution

$$f(x) = \alpha k^\alpha x^{-\alpha-1}, x \geq k; \alpha, k > 0;$$

$$E[X] = \begin{cases} \frac{k^2 \alpha}{\alpha - 2}, & \alpha > 2, \\ \infty, & \alpha \leq 2 \end{cases}$$

- Series System MTTF is
$$\frac{1}{\sum_{i=1}^{n} \lambda_i}$$
- Parallel System
$$R_X(t) = 1 - (1 - e^{-\lambda t})^n$$
MTTF of a parallel redundent system is given by
$$E[X] = \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{i} = \frac{H_n}{\lambda} \simeq \frac{ln(n) + C}{\lambda}$$
- Standby Redundancy
$$E[X] = \frac{n}{\lambda} \text{ and } Var[X] = \frac{n}{\lambda^2}$$
- TMR and TMR/Simplex Systems
MTTF of a TMR system is
$$E[X] = \frac{5}{6\lambda}$$
MTTF of aTMR/Simplex system is
$$E[X] = \frac{4}{3\lambda}$$