Topics Covered in Lecture 14

29-August, 2002

- Home work # 2 solutions are discussed.
- Transform Methods
  For a random variable X, $e^{X\theta}$ is another random variable. The expectation $E[e^{X\theta}]$ will be a function of θ. The moment generating function (MGF) is
  $$M(\theta) = E[e^{X\theta}]$$
  $$M(\theta) = \begin{cases} \sum_j e^{x_j\theta} p(x_j), & \text{if X is descrete,} \\ \int_{-\infty}^{\infty} e^{x\theta} f(x)dx, & \text{if X is continuous} \end{cases}$$
  Laplace-Stieltjes transform (LST) of X
  $$L_X(x) = L(s) = M_X(-s) = \int_0^{\infty} e^{-sx} dF(x) = \int_0^{\infty} e^{-sx} f(x)dx.$$
- Properties of Transforms
  - $$Y = aX + b$$
    $$L_Y(s) = e^{-bs} L_X(as)$$
  - $$Z = X + Y$$
    $$L_Z(s) = L_X(s).L_Y(s) \qquad \text{assuming X and Y are independent}$$
  - $$\begin{aligned} L_X(s) &= E[e^{-sX}] \\ &= E\left[1 - sX + \tfrac{s^2X^2}{2!} - \ldots\right] \\ &= 1 - sE[X] + \tfrac{s^2E[X^2]}{2!} - \ldots \end{aligned}$$

Substitute s = 0 then

$$L_X(0) = 1$$

$$\frac{dL_X(s)}{ds} = -E[X] + 2sE[X]^2 - \dots$$

Substitute s = 0 then

$$\frac{dL_X(0)}{ds} = -E[X]$$

$$E[X] = -\frac{dL_X(0)}{ds}$$

- Z = X + Y, X $\sim$ EXP(λ_1) and Y $\sim$ EXP(λ_2)

$$\begin{aligned} L_Z(s) &= L_X(s), L_Y(s) \\ &= \left(\tfrac{\lambda_1}{\lambda_1+s}\right)\left(\tfrac{\lambda_2}{\lambda_2+s}\right) \end{aligned}$$