Topics Covered in Lecture 12

22-August, 2002

- Chapter 4 titled **Expectation** is started.
- **Definition(Expectation)**. The expectation, E[X], of a random variable X is defined by

$$E[X] = \begin{cases} \sum_i x_i p(x_i), & \text{if X is descrete,} \\ \int_{-\infty}^{\infty} x f(x)\, dx, & \text{if X is continuous,} \end{cases}$$

- If we are searching for an element in an array, then

$$p_X(i) = \frac{1}{n}, 1 \leq i \leq n.$$

$$E[X] = \sum_{i=1}^{n} i p_X(i) = \frac{(n+1)}{2}.$$

- **Definition(Zipf's law)** p_Y(i), the probability of a request for the i the most popular page is inversely proportional to i.

$$p_Y(i) = \frac{c}{i}, 1 \leq i \leq n,$$

$$\sum_{i=1}^{n} \frac{c}{i} = 1$$

$$c.H_n = 1$$

$$c \simeq \frac{1}{(\ln(n) + C)}$$

$$p_Y(i) = \frac{1}{iH_n}$$

$$E[Y] = \sum_{i=1}^{n} i p_Y(i) = \sum_{i=1}^{n} \frac{1}{H_n} = \frac{n}{H_n}$$

If we assume that the "least frequently used", cache replacement policy is used then hit ratio $h(m)$ is given by.

$$h(m) = \sum_{i=1}^{m} p_Y(i) \simeq cH_m = \frac{H_m}{H_n} \simeq \frac{\ln(m) + C}{\ln(n) + C}$$

- Let X be a continuous random variable with $X \sim EXP(\lambda)$

$$f(x) = \lambda e^{-\lambda x}, \qquad x > 0.$$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx = \int_{0}^{\infty} \lambda x e^{-\lambda x} dx$$

Let $u = \lambda x$, then $du = \lambda dx$, and

$$E[X] = \frac{1}{\lambda} \int_{0}^{\infty} u e^{-u} du = \frac{1}{\lambda}$$

- Moments

$$Y = \phi(X)$$

$$E[Y] = E[\phi(X)] = \begin{cases} \sum_i \phi(x_i) p_X(x_i), & \text{if X is descrete,} \\ \\ \int_{-\infty}^{\infty} \phi(x) f_X(x) dx, & \text{if X is continuous,} \end{cases}$$

- $E[X^k]$ is known as the kth moment of the random variable X.
- **Definition(Variance)** The variance of a random variable X is

$$Var[X] = \mu_2 = \sigma_X^2 = \begin{cases} \sum_i (x_i - E[X])^2 p(x_i) & \text{if X is descrete,} \\ \int_{-\infty}^{\infty} (x - E[X])^2 f(x) dx & \text{if X is continuous} \end{cases}$$

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$$\phi(X) = aX + b$$

$$E[aX + b] = a.E[X] + b$$

$$\begin{aligned} E[(X - E[X])^2] &= E[X^2 + (E[X])^2 - 2XE[X]] \\ &= E[X^2] - (E[X])^2 \end{aligned}$$