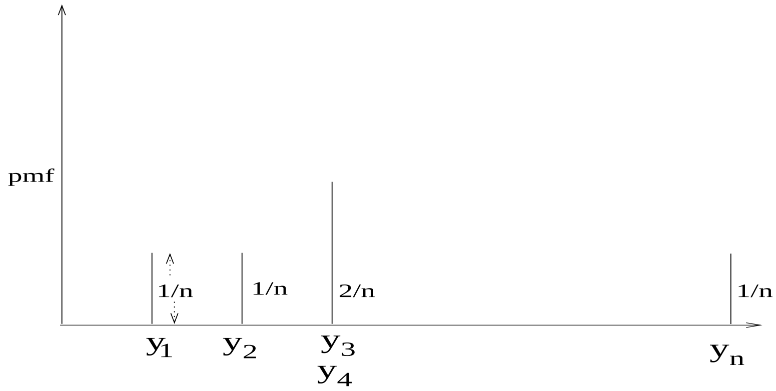
Topics Covered in Lecture 9

19-August, 2002

- To generate the random diviate of the EXP(λ) r.v., first generate a random number u, random deviate of U(0, 1) r.v., and then apply the formula,
$$x = -\frac{ln(1-u)}{\lambda}$$
- If we apply the procedure repeatedly to obrain a sequence of n random diviates: $x_1, x_2, x_3, \ldots, x_n$ and order them into a sequence of increasing values $y_1 \leq y_2 \leq y_3 \ldots \leq y_n$. Plot the emperical pmf on histogram. and emperical cdf from these. For large n, the emperical cdf should approach EXP(λ)theoritical cdf.



- More generally take the CDF of some r.v.$F_X(x)$ and take the inverse of $F_X^{-1}(u)$ for a random number u to obrain random diviate of X. This applies when F_X is invertiable(e.g.EXP(λ)).
- For Weibull distribution
$$u = 1 - e^{-\lambda x^{\alpha}} \Rightarrow 1 - u = e^{-\lambda x^{\alpha}}$$
$$x = \left[-\frac{ln(1-u)}{\lambda}\right]^{1/\alpha}$$

- X $\sim$ EXP(λ)
  Y = rX

  $$P(Y \leq y) = P(rX \leq y) = P(X \leq y/r) = e^{-\lambda y/r} = e^{-(\lambda/r)y}$$

  $\Rightarrow$ Y $\sim$ EXP(λ/r)
  EXP is closed under multiplication by constant (positive real constant).

- X $\sim$ N(μ, σ^2)   Y = e^X
  Y is lognormal (repair times are often found to obey the lognormal distribution)

- two r.v. defined on the same sample space, X and Y

  $$F_{X,Y}(x, y) = P(X \leq x \text{ and } Y \leq y) = p(\{x \in S | X(s) \leq x, Y(s) \leq y\})$$

  1. F(x, y) $\in$ (0, 1)
  2. Monotonic in x and y
  3. $$\lim_{x \to -\infty} F(x, y) = \lim_{y \to -\infty} F(x, y) = 0$$
     $$\lim_{x \to \infty} \lim_{y \to \infty} F(x, y) = 1$$
  4. F(x, y) is right continuous ingeneral and continuous if both X and Y are continuous..
     $$\lim_{y \to \infty} F(x, y) = F_X(x), \text{ the marginal distribution of X}$$

- Order statistics