Topics Covered in Lecture 8

14-August, 2002

- Bathtub shap curve
- Inverse bathtub shape curve
- Hypoexponential distribution: If a process consists of several phases and if each phase follows exponential distribution function then to model such kind of systems this distribution is used.

$$f(t) = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

- Erlang distribution: In previous distribution if $\lambda_1 = \lambda_2$ then it is called as Erlang distribution.

$$\text{In r-phase Erlang distribution} f(t) = \frac{\lambda^r t^{r-1} e^{-\lambda t}}{(r-1)!}$$

$$F(t) = 1 - \sum_{k=0}^{r-1} \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

- Gamma distribution

$$f(t) = \frac{\lambda^\alpha t^{\alpha-1} e^{-\lambda t}}{\Gamma(\alpha)}, \quad \alpha > 0, t > 0$$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx \quad \alpha > 0$$

$$\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\int_0^\infty x^{\alpha-1} e^{-\lambda x} dx = \frac{\Gamma(\alpha)}{\lambda^\alpha}$$

- X $\sim$ GAM(1/2, n/2) is Chi-Square distribution with n degrees of freedom.

- Hyperexponential distribution

$$f(t) = \sum_{i=1}^{k} \alpha_i \lambda_i e^{-\lambda_i t}, \quad t > 0, \lambda_i > 0, \alpha_i > 0, \sum_{i=1}^{k} \alpha_i = 1$$

$$F(t) = \sum_{i=1}^{k} \alpha_i \left(1 - e^{-\lambda_i t}\right), \quad t \geq 0$$

- Weibull distribution

$$f(t) = \lambda \alpha t^{\alpha-1} e^{-\lambda t^{\alpha}}$$
$$F(t) = 1 - e^{-\lambda t^{\alpha}}$$
$$h(t) = \lambda \alpha t^{\alpha-1}$$

- Log-Logistic distribution:

$$f(t) = \frac{\lambda \kappa (\lambda t)^{\kappa-1}}{\left[1 + (\lambda t)^{\kappa}\right]^2}$$

$$F(t) = 1 - \frac{1}{1 + (\lambda t)^{\kappa}}$$

$$h(t) = \frac{\lambda \kappa (\lambda t)^{\kappa-1}}{1 + (\lambda t)^{\kappa}}$$

when $\kappa \leq 1$ it becomes DFR
when $\kappa > 1$ it becomes Inverse bathtub

- Normal or Guassian distribution

- The uniform or Rectangular distribution

- Pareto distribution

- Defective distribution

- functions of a random variable