Assignment 09

This assignment consists of 6 problems. Problems 7, 8 and 9 are from 475 - 476 pages. Problems 1, 2 and 3 from 482 - 483 pages.

Problem #7:
For the preventive maintenance problem (Example 8.20), derive an expression for the optimal inspection rate λ_{in}^* so as to maximize the steady-state availability.

Problem #8:
Consider an approximation for the CTMC model of Example 8.21 where the delay state is replaced by an instantaneous branch point. Using the technique of Example 5.4 or using equation(5.30). Show that the coverage probability is $\delta/(\delta+\lambda)$. Solve the reduced model and obtain an expression for the error of approximation in steady-state availability.

Problem #9:
Modify and solve the CTMC of Example 8.23 so as to instroduce transient faults that are treated by a retry. Resulting CTMC will have an arc leading from state $1D$ to state 2 at the rate δr where r is the probability fo transient restoration. Clearly, the arc from state $1D$ to $1C$ will now be relabelled as $\delta(1-r-c)$.

Problem #1:
Consider the following concurrent program [TOWS 1978] with a cyclic structure:

repeat
 TCPU1;
 if B then TIO1,
  else
  cobegin
   TCPU2; TIO2
  coend

forever.

Assume that successive tests on condition B form a sequence of Bernouli trials with probability of failure q. The execution times of the statement groups (or tasks) TCPU1 and TCPU2 are EXP(μ_1) and EXP(μ_2) random variables, respectively, while the execution times of TIO1 and TIO2 are both EXP(λ) random variables. Draw the CTMC state diagram of this system and solve for the steady-state probabilities. Assuming that TCPU1 and TCPU2 are executed on a single CPU and that TIO1 and TIO2 are executed on a single I/O processor, compute steady-state utilizations of the two processors. Use $1/\mu_1 = 8$ms, $1/\mu_2 = 26.6$ms, $1/\lambda = 46.1$ms, and vary q from 0 to 1.

Problem #2:

Suppose we want to purchase a two-processor system, to be operated as an $M/M/2$ queue. Keeping the total amount of computing power constant, we want to investigate the tradeoffs between a system with homogeneous processors (total power 2μ) and a heterogeneous system (total power $\mu_1 + \mu_2 = 2\mu$). Using the average response time as the criterion function, which system will be superior? To take a concrete case, let $\rho = \lambda/2\mu$ vary from 0.1 to 0.9 and compute the average response times of the two ssytems, choosing different sets of values of μ_1 and μ_2 and assuming $\lambda = 1$ [It can be shown that the optimum values of μ_1 and μ_2 are given by

$$\lambda\left(\frac{1}{\rho} + 1 - \sqrt{1 + \frac{1}{rho}}\right) \text{ and } \lambda\left(\sqrt{1 + \frac{1}{\rho}} - 1\right) \text{ where } \rho = \frac{\lambda}{2\mu}$$

Problem #3:

Write a discrete-event simulation program to simulate the $M/M/2$ heterogeneous queuing system with $\mu_2 = 0.25$, $\lambda = 0.2$, and α varied as in Table 8.5. In order to estimate the steady-state response times $E[R_1], E[R_2]$, and $E[R]$ as defined in Table 8.5 , you have to execute three different simulation (two for an $M/M/1$ queue and one for the $M/M/2$ case), discard the statistics corresponding to initial transients, and then collect the steady-state values. The attainment of the steady state is determined by experimentation. For statistical analysis of outputs, see Chapter 10 of this book.