Assignment 07

This assignment consists of 7 problems. Problems 1, 7 from 420 - 421 pages. Problems 1, 2 of 425 - 426 pages and Problems 1, 3, 4 of 435 - 436 pages.

Problem #1:
Consider an $M/M/1$ queue with an arrival rate λ and the service rate μ. We have derived the distribution function of the response time R. Now we are interested in deriving the distribution function of the waiting time W. The waiting time W is the response time minus the service time. To get started, first compute the conditional distribution of W conditioned on the number of jobs in the system, and later compute the unconditional distribution function. Note that W is a mixed random variable since its distribution function has a jump equal to $P(W = 0)$ at the origin.

Problem #7:
Define the perceived mean queue length [GEIS 1983] $N^* = E[N^2]/E[N]$ and the perceived mean response time by $R^* = E[R^2]/E[R]$. Derive formulae for N^* and R^* in the $M/M/1$ queue.

Problem #1:
Consider a telephone switching system consisting of n trunks with an infinite caller population. The arrival stream is Poisson with rate λ and call holding times are exponentially distributed with average $1/\mu$. The traffic offered, A(in Erlangs), is defined to be the average number of call arrivals per holding time. Thus, $A = \lambda/\mu = \rho$, the traffic intensity. We assume that an arriving call is lost if all trunks are busy. This is known as BCC(blocked calls cleared) scheduling discipline. Draw the state diagram and derive an expression for π_i, the steadystate probability that i trunks are busy. Show that this pmf approaches the Poisson pmf in the limit $n \to \infty$(i.e., ample-trunks case). Therefore, for finite n, the above pmf is known as the "truncated Poisson pmf". Define the **call congestion**, $B(n)$, as the proportion of lost calls in

the long run. Then show that

$$B(n) = \frac{\frac{\rho^n}{n!}}{\sum_{i=0}^{n} \frac{\rho^i}{i!}}.$$

This is known as Erlang's B formula. Define traffic carried, C (in Erlangs), to be the average number of calls completed in a time interval $1/\mu$. Then

$$C(n) = \sum_{i=0}^{n} i\pi_i.$$

verify that:

$$B(n) = 1 - \frac{C(n)}{A}$$

Show that the efficient formula to compute the loss probability holds [AKIM1993]:

$$B(k) = \frac{\frac{\rho}{k}B(k-1)}{1 + \frac{\rho}{k}B(k-1)}, k = 1, 2, \ldots, n.$$

where $B(0) = 1$.

Problem #2:
Derive the steady-state distribution of the waiting time W for an $M/M/2$ queuing system as follows:

- First show that $P(W = 0) = \pi_0 + \pi_1$.
- Now, assuming that n $\geq$ 2 jobs being present in the system at the time of arrival of the tagged job, argue that the distribution of W is $(n-1)$-stage Erlang with parameter 2μ.

Compute the distribution function and hence compute the expected value of W.

Problem #1:
Plot the response time distribution (both conditional and unconditional) of the $M/M/1/n$ queue assuming $\lambda = 0.9$ and $\mu = 1$. Vary $n = 10$, 50, 100. Also plot the rejection probability π_n, as a function of n. Obtain the expression for the mean response time from the (conditional) response time distribution (equation(8.56)) and show that it is the same as that obtained using Little's result.

Problem #3:

Plot task completion time distributions for the two-state availability model for the two cases of $\lambda = 0$ and $\lambda = 0.1$. Assume $\mu = 0$ and $x = 10$. For extra credit, numerically invert the LSTs for the *prt* cases and plot these two distribution functions as well on the same plot. Assume $\mu = 1$ and $\lambda = 0.1$.

Problem #4:
From the LSTs of the completion time (equation(8.60) and (8.61)), derive expressions for the average completion time for the *prs* and *prt* cases in the two-state availibility model.