Assignment 04

This assignment conssits of 5 problems. Problem 3 is from 165 page. Problems 4, 8 of 166 page and problem 4 of page 197. Problem 5 from 207 page.

Problem #3:
Consider a series system consisting of n independent components. Asuming that the lifetime of the ith component is Weibull distributed with parameter λ_i and α, show that the system lifetime also has a Weibull distribution.
As an concrete example, consider a liquid cooling catridge system that is used in enterprise-class servers made by Sun Microsystems [KOSL 2001]. The series system consists of a blower, a water pump and a compressor. The following table gives the Weibull data for the three components.

$Component$	$L10(h)$	$Shape.parameter\ (\alpha)$
$Blower$	$70,000$	3.0
$Waterpump$	$100,000$	3.0
$Compressor$	$100,000$	3.0

L10 is the *rating life* of the component, which is the time at which 10 % of the components are expected to have failed or R(L10) = 0.9. Derive the sytem reliability expression.

Problem #4:
Consider a random access memory card consisting of d VLSI chips, each containing w bits. Each chip supplies one bit position in a d-bit word for a total of w, d-bit words. Assuming a failure rate of λ per chip(and an exponential lifetime distribution), derive the reliability expression $R_0(t)$ for the memory card. Suppose that we now introduce single error correction, so that up to one chip may fail without a system failure. Note that this will require c extra chips where c must satisfy the relation $c \geq \log_2(c + d + 1)$. (See Rao and Fujiwara [RAO 1989].) Derive the reliability expression $R_1(t)$. Plot $R_0(t)$ and $R_1(t)$ as functions of t on the same plot. Derive expressions

for the hazard functions $h_0(t)$ and $h_1(t)$ and plot these as functions of time t. For these plots assume $d = 16$ (hence $c = 5$) and $\lambda = 10$ per million hours.

Problem #8:
Consider a system with n independent components each having the modified exponential lifetime distribution with initial mass p_0 at origin and rate λ [see equation (3.2)]. derive reliability expression for a k-out-of-n system and specialize to the cases of $k = 1$(parallel) and $k = n$(series).

Problem #4:
For the modified exponential distribution with a mass at origin [formula (3.2)], compute its expected value.

Problem #5:
A certain telephone company charges for calls in the following way: \$0.20 for the first 3 min or less; \$0.08 per min for any additional time. Thus, if X is the duration of a call, the costs Y is given by

$$Y = \begin{cases} 0.20, & 0 \leq X \leq 3, \\ 0.20 + 0.08(X - 3), & X \geq 3. \end{cases}$$

Find the expected value of the cost of a call (E[Y]), assuming that the duration of a call is exponentially distributed with a mean of $1/\lambda$ minutes. Use $1/\lambda =$ 1, 2, 3, 3, 5 min.