



Euclidean distance for  $k=7$

As observed from the residual variance curve, the dimensionality is 1. Let the angles be  $\theta_1, \theta_2, \theta_3$  and  $\theta_4$  as mentioned in the pdf file of Part B3. As observed in the image files, the box remains at a constant height above the base of the two arms. If there was no box and the arms could move independently, the degree of freedom would have been 4 as in Part B. But since the height of the box has to be constant, we conclude that  $l_1 \sin\theta_1 + l_2 \sin\theta_2 = \text{constant}$  where  $l_1, l_2, l_3, l_4$  are the length of the arms making the corresponding  $\theta$ 's. Therefore taking  $\theta_1$  to be the independent variable,  $\theta_2$  depends on  $\theta_1$ . Similarly, for arm2,  $\theta_4$  depends on  $\theta_3$ . The motion of the arms is constrained along the horizontal direction by the equation

$$l_3 \cos \theta_3 + l_4 \cos \theta_4 - l_1 \sin\theta_1 + l_2 \sin\theta_2 = \text{constant} = \text{width of the box}$$

as the two arms always hold the box along the midline as seen in the images. Therefore,  $\theta_3$  depends on  $\theta_1$ . Therefore taking  $\theta_1$  as the independent variable, all other variable can be expressed in terms of  $\theta_1$  reducing the dimensionality of the problem to 1.