Power Allocation Issues in a Wireless Mesh Network

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Abstract

Spatial-reuse Time Division Multiple Access or STDMA is a MAC technique that allows simultaneous reception and simultaneous transmission of signals in a wireless network. Simultaneous transmission and reception between different antennas that are geographically very close impose various constraints on the power transmitted by these antennas. This report will delve into how to determine a feasible power allocation to the various antennas in a given topology. We focus on an 802.11-based wireless mesh network built using point-to-point links.

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1 Introduction

The Digital Gangetic Plains is an ongoing IEEE802.11b testbed, that has successfully used off-the-shelf 802.11b indoor equipment and high gain directional antennae to create a large outdoor multi-hop network spanning upto 80 kilometers [1]. The 802.11b equipment implements a CSMA-based MAC protocol and is primarily meant for indoor usage. With more than two people transmitting in an indoor environment, both transmissions interfere with each other quite significantly and communication will not take place. This however is not the case when it comes to outdoor networks. It can be shown [2] that with a sufficient angle of separation between two directional antennae, a node can simultaneously receive from or transmit to more than one of its neighbours. This sets the stage for 2-phase STDMA [2] as a potentially better protocol than the contention-based CSMA for such networks. In a 2-phase STDMA protocol, each node (which is a collection of various antenna at a given tower, let's say), have 2 phases, a transmit phase and a receive phase. In the transmit phase, all the antennas transmit data along their links and once the phase is over, they all switch over to receive phase, where they receive data from their respective links.

While it has been shown that the angle of separation of directional antennae is an important factor in determining whether or not two different antennae at a fixed node(antenna) can receive transmissions simultaneously, another factor that is known to be important is the allocation of transmit powers for each antenna. We are studying the various intricacies in allocating power for antennae in different topologies given that they use the STDMA protocol.

Figure 1 depicts an example topology to illustrate certain issues in power allocation. The gain of the antenna on the $3.2km$ link is $10dB$ while the gain of the antenna on the $25km$ link is $24dB$. Both these links will operate if run individually, but when they operate together the transmission of the $3.2km$ link interferes with that of the $25km$ link, and the signal that is received at the antenna is not strong enough for a meaningful transmission. It is possible to allocate power to this topology if we increase the gain of the antennae on the shorter link but that may not be viable

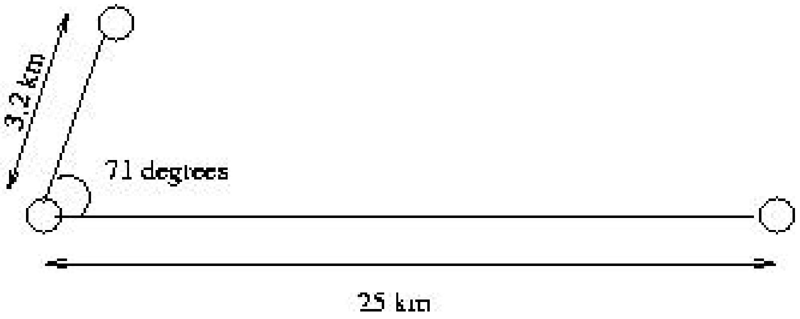

Figure 1: An Example of Power Allocation Complications

economically. This shows that interference can play a role in the allocation of power to different nodes in a topology.

Background on Wireless Transmission and Propagation

Some important equations that help us analyse power constraints and complexities in wireless transmissions:
The free-space pathloss in dB for a transmission at f MegaHertz between two points that are d kilometers apart is

$$32.5 + 20 * \log d + 20 * \log f \tag{1}$$

Therefore, if the two antennae have directional gains of G_1 and G_2, then the power received by one if the other transmits with power T would be

$$T + G_1 + G_2 - (32.5 + 20 * \log d + 20 * \log f) \tag{2}$$

An antenna can correctly receive a signal P despite the occurence of any interfering signal I if

$$P - I \geq SIR_{reqd} \tag{3}$$

where SIR_{reqd} is the minimum signal to interference ratio required for valid transmission and is a function of the bitrate. This has been experimentally measured to be around $10 - 13dB$ for 11 Mbps transmissions [2]. It is important to note that all these equations hold in dB space.

Characteristics of topologies under consideration

A "node" can be defined as a collection of antennae at a fixed point. This would be a fixed tower with different antennae pointing at different directions. In the 2-phase STDMA scheme proposed [2], for any given topology, nodes will alternate between their transmitting and receiving phases. This is possible if the given topology is a bipartite graph where any node can have multiple edges to nodes not in the same partition. All nodes in the same partition will simultaneously receive or send at the same time. In this report we shall refer to such topologies as STDMA topologies. We will consider only bipartite graphs as it corresponds to a single transmit and receive phase and allows the 2-phase STDMA protocol.

In Figure 2, we denote the bipartite classes as '1' and '0'. **Figure A** has two nodes in the same bipartite class as neighbours with a link between them and is hence an invalid STDMA topology. **Figure B** denotes a path and is a valid STDMA topology. **Figure C** and **Figure D** are sample STDMA topologies of maximum degree 2 and 3 respectively.

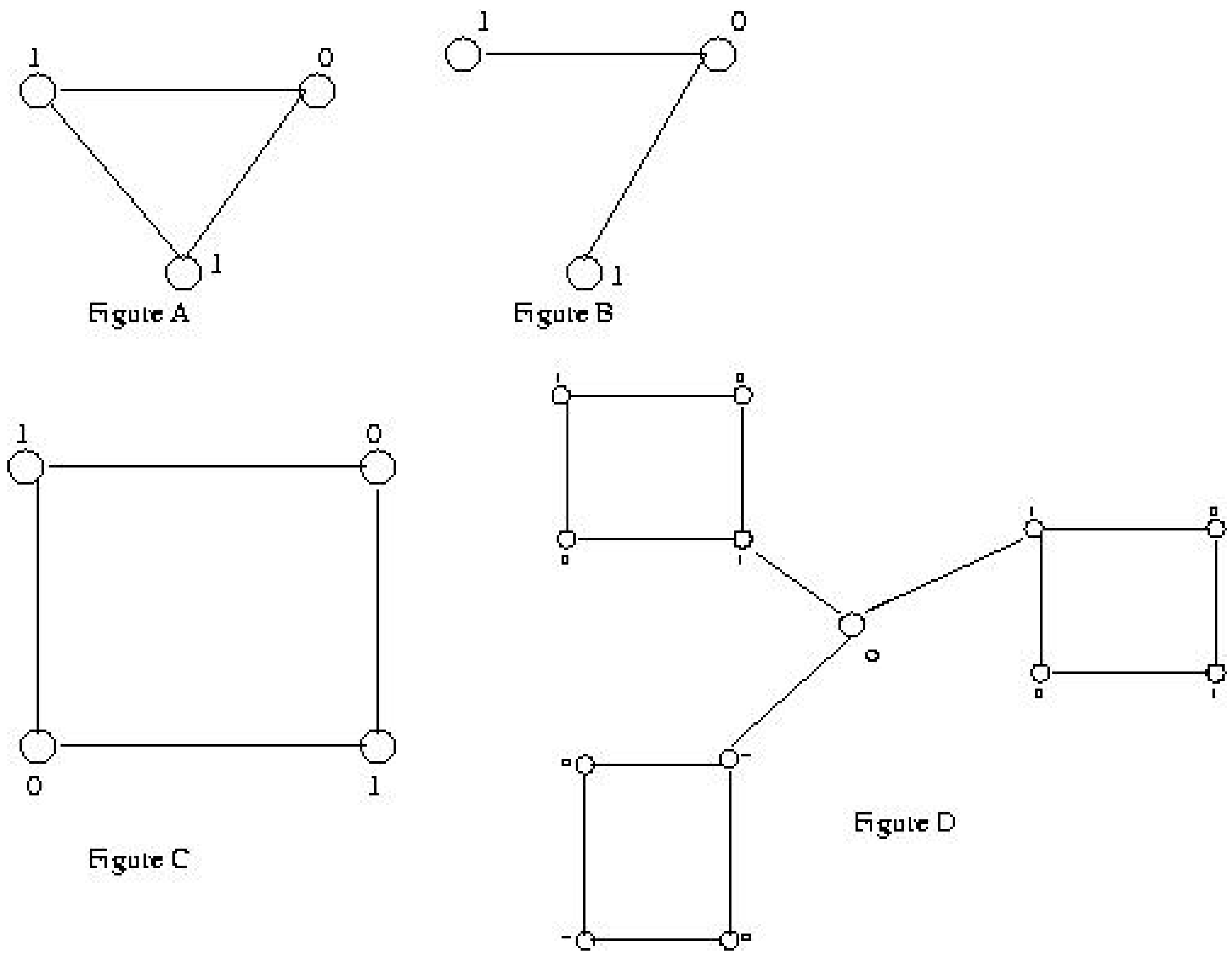

Figure 2: Some Sample Topologies

We initially decided to come up with a model that can express power requirements for a topology which is either a cycle or a path. In section 2 we outline the model we devised, followed by section 3 which is able to express the power requirements for topologies of any degree. This model came at the cost of having to very conservatively estimate the interference in the network. This is rectified in the model described in section 4. This is followed by some design considerations and a results section.

2 Cycles and Paths

Initially we decided to concentrate on topologies where the maximum degree of a given node is 2. In [2], a formulation of a channel allocation problem is specified and heuristics which can convert graphs of maximum degree upto 5 into channel subgraphs which will allow for minimum interference. The resulting channel subgraphs are of degree 2, that is, paths or even cycles. Hence we decided to focus on such graphs. We defined a formalism to represent such a graph and derived a set of equations and inequalities that would have to hold inorder to have a consistent power allocation throughout the graph.

Nodes are denoted a_i ranging from 0 to $N-1$
$P_{i,0}$ = power received at a_i from to a_{i-1}
$P_{i,1}$ = power received at a_i from to a_{i+1}

$T_{i,0}$ = power transmitted from a_i to a_{i-1}
$T_{i,1}$ = power transmitted from a_i to a_{i+1}
P_{min} = minimum power that can be received
T_{max} = maximum power that can be transmitted
L_i is 1 if the link between a_i and a_{i+1} exists and 0 otherwise
d_i is the distance between nodes a_i and a_{i+1}
SL_i is the side lobe below the main direction for the two antennas at node a_i
(depending on the mutual angle of the two antennas
SIR_{req} is the minimum Signal to Interference ratio below which transmission won't be heard

Some assumptions that are inherent in this formalism are

- There is no information about distance of a node from other nodes apart from its neighbours
- No information regarding the Gain observed between two non-neighbouring nodes can be obtained

The following two equations state that the power transmitted along with the gain induced by the antenna should not exceed the maximum transmit power that is allowed. L_i guarantees that the LHS will be zero if the link does not exist. Maximum transmit power may vary from place to place depending on court laws governing wireless transmission.

$$(G_{i,0} + T_{i,0}) * L_{i-1} \leq T_{max} \tag{4}$$

$$(G_{i,1} + T_{i,1}) * L_i \leq T_{max} \tag{5}$$

The following two equations calculate the power received at any antenna as per our convention. This can be seen to be in conformance with equation 2. The L_i term guarantees that $P_{i,1}$ will be 0 if the corresponding link is down.

$$P_{i,0} = [T_{i-1,1} + G_{i-1,1} + G_{i,0} - (32.5 + 20 \log d_{i-1} + 20 \log f)] * L_{i-1} \tag{6}$$

$$P_{i,1} = [T_{i+1,0} + G_{i+1,0} + G_{i,1} - (32.5 + 20 \log d_i + 20 \log f)] * L_i \tag{7}$$

The following two equations state that the power received at any antenna should be greater than the minimum threshold power which an antenna can receive.

$$P_{i,0} \geq P_{min} * L_{i-1} \tag{8}$$

$$P_{i,1} \geq P_{min} * L_i \tag{9}$$

The following two equations indicate that the difference in power received at the two antennas at a given node does not exceed the Signal to Interference ratio. The additional 3db term is added to take into consideration the interference due to the transmissions of other antennae at the neighbouring nodes but whose presence might be felt[1] at the node under consideration. These equations can be seen to be in conformance with equation 3.

$$L_i L_{i-1} * (P_{i,0} - (P_{i,1} + 3 * L_{i-2} - SL_i)) \geq L_i L_{i-1} * SIR_{reqd} \tag{10}$$

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[1] We "assume" that only the antennae on the node which is transmitting to the antenna under consideration can transmit an interfering signal, whereas all nodes in the graph that transmit at that time can cause an interfering signal. We are only considering those nodes that are one hop away from the node in consideration. This means that if we are considering the transmission between $a_{i-1,1}$ to $a_{i,0}$, then the only two interfering signals would be from $a_{i+1,0}$ and $a_{i-1,0}$.

$$L_iL_{i-1} * (P_{i,1} - (P_{i,0} + 3 * L_{i+1} - SL_i)) \geq L_iL_{i-1} * SIR_{reqd} \tag{11}$$

Like the above two equations, the following two equations are identical, but take care of the case where the interference due to the transmission of other antennae at the neighbouring nodes is greater than the interference due to the reception of the other antenna present at node a_i.

$$L_{i-2}L_{i-1} * [P_{i,0} - (T_{i-1,0} + G_{i-1,0} - SL_{i-1} - (32.5 + 20\log d_{i-1} + 20\log f) + 3 * L_i)] \geq SIR_{reqd} \tag{12}$$

$$L_iL_{i+1} * [P_{i,1} - (T_{i+1,1} + G_{i+1,1} - SL_{i+1} - (32.5 + 20\log d_i + 20\log f) + 3 * L_{i-1})] \geq SIR_{reqd} \tag{13}$$

Each of equations 10, 11, 12, 13 have a term 3 which is added to the interference of the corresponding signal. These equations are in dB-space and adding a $3dB$ term corresponds to multiplying the term by 2 in physical terms. As interference is additive in physical space, we add a $3dB$ term which corresponds to doubling the interference of the corresponding signal. We assume that there are a maximum of two interfering signals and ideally we would like to add them to find the resultant power of the interfering signal. As we operate in a dB-space, if we were to linearly add the interference we would no longer have linear inequalities as above. Therefore we add a 3dB term as a conservative measure to take twice the power of the interfering signal which would then accomodate the effect of the other interfering signal also.

Now we have a system of linear equations which can categorize a fixed set of topologies. There are various methods to solve systems of linear equations and the above equations were coded and solved using Matlab. We tested our program on a few simple topologies and we were able to attain feasible power allocation for the nodes.

We now know that we can use linear programming to solve the problem of power allocation in-order to be compliant with the STDMA protocol. Our results our optimal for the set of constraints given(as expected) but these are still to be substantiated with field data. The current set of equations leave a lot to be desired both in terms of the assumptions made regarding interference signals and in terms of flexibility in defining topologies.

3 Arbitrary Topologies

We were then able to come up with a far simpler, but more expressive set of equations which are able to handle all STDMA topologies, and eliminate the drawbacks of the previous method. We then implemented these set of equations and were able to verify it successfully on various topologies.

We now allow a node to have any degree and number the antennae from 1 to N keeping information about its position (co-ordinates) and its neighbour with whom it communicates. It is important to note that all antennae with the same position would constitute a node. As this is an STDMA graph the antennae can be partitioned into two sets that are bipartite. We store this information also.

Antennae are a_i
P_i = power received at a_i from the neighbour it communicates with
$P_{i,j}$ = power received at a_i from to a_j
T_i = power transmitted by a_i

The equations can now be rewritten as

$$T_i + G_i \leq T_{max} \forall i \tag{14}$$

$$P_i \geq P_{min} \forall i \tag{15}$$

$$P_i - (P_{i,k} + 10 \log (\frac{N}{2} - 1)) \geq SIR_{reqd} \forall i, k \tag{16}$$

In the above equation k stands for all those antennae $a_k$which are not in the same bipartite partition as a_i and not equal to a_i's neighbour. The term $10 \log (N/2 - 1)$ stands for the conservative upper bound for the combined interference of all the transmissions taking place at a single instant.

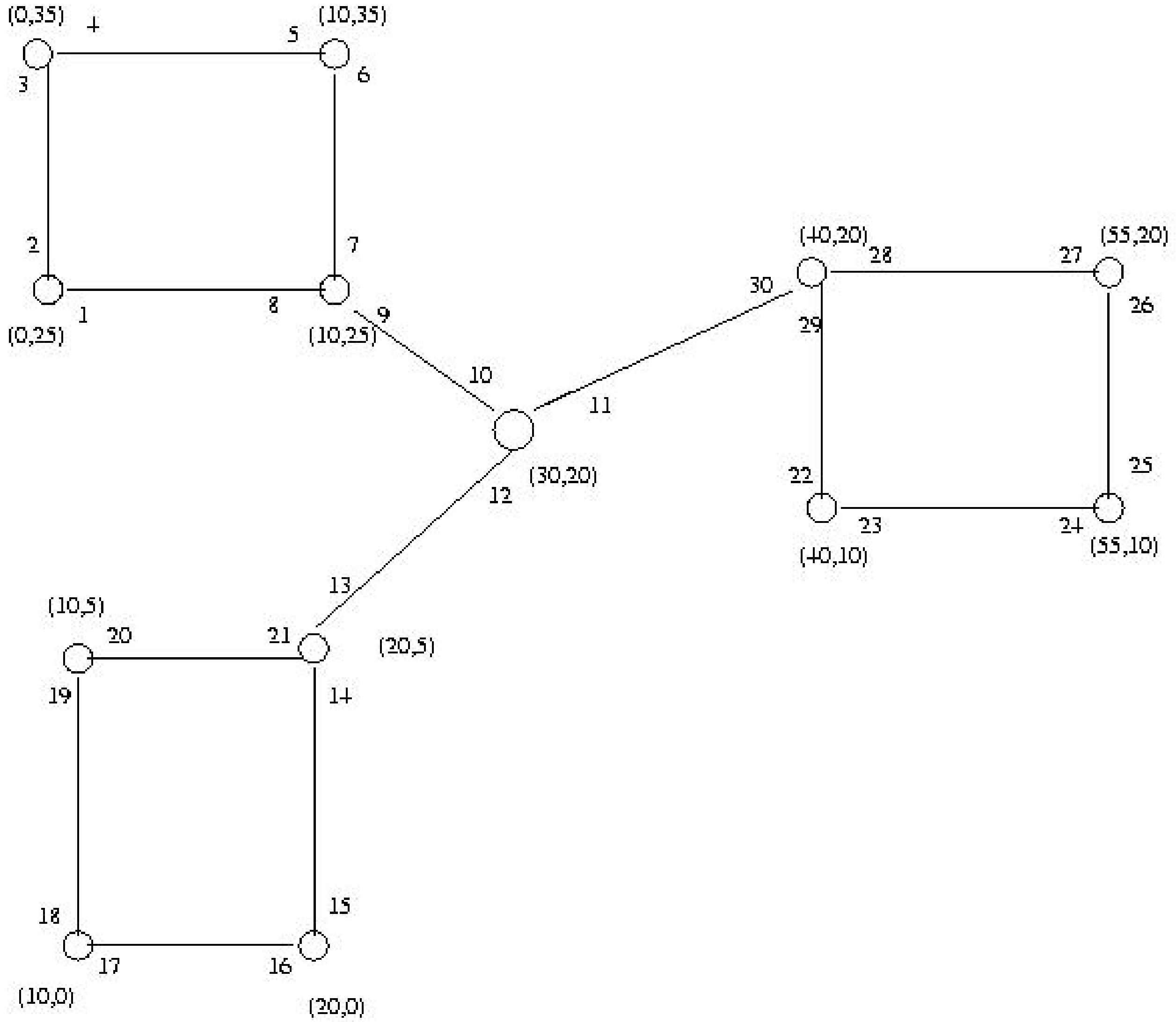

Figure 3: Sample Topology and Power Allocation

We were successfully able to allocate power to the topology depicted in Figure 3 which contains 13 nodes, 15 links and 30 antennae. The degree of this topology is 3, and hence does not fall under the classification of a cycle or path. This network spans $55km$ in length and $35km$ in breadth. The numbers (x, y) represent the cartesian coordinates of the nodes while the other numbers denotes the antenna-id[2]. The unit of distance is kilometers. For this fairly complex topology we were able to generate the power allocation as shown in Table 1. which is consistent with the power equations and can hence use the 2-phase STDMA protocol.

We have implemented the above in Matlab and have successfully allocated power to various topologies. Although we were able to get some reasonable power allocations for some topologies we feel

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[2] the antenna-id is the same as i for antenna a_i

Antenna ID	**Power Allocated (dBm)**
1	-12.8958
2	-12.8958
3	-12.8958
4	-12.8958
5	-12.8958
6	-12.8958
7	-11.7977
8	-7.0043
9	-12.6119
10	-10.2076
11	-7.0043
12	-7.7769
13	-7.7769
14	-18.9164
21	-12.8958
15	-18.9164
16	-12.8958
17	-12.8958
18	-18.9164
19	-11.7977
20	-12.8958
22	-12.8958
23	-9.3739
24	-9.3739
25	-12.8958
26	-12.8958
27	-15.3739
28	-10.2076
29	-12.8958
30	-12.8958

Table 1: Results of Power Allocation Program

that interpolating the interference using the $10 \log (N/2 - 1)$ term is too stringent a constraint, as we assume much more interference than there will actually exist and we will have to come up with a better method for modeling interference.

We now have a system where a user can input a topology and has the flexibility to choose different frequencies of communication for different links, different gains for various antennae and any number of antennae at a given node. We have successfully used linear programming as a tool to calculate feasible power allocations for various topologies. In our implementation, the user has to specify all characteristics of every link in the topology and our program finds a feasible power allocation if it exists. We will change the implementation such that the user only has to input the topology and no link characteristics unless he specifically wishes to do so.

4 Exact Interference Model

We implemented a model that takes into account the exact interference seen at an antenna based on all other transmissions happening in the network. The previous models used terms like $3dB$ and $10 \log N/2 - 1$ to conservatively estimate the interference. Let $G_{i,j}$ denote the gain when antenna a_i is transmitting to antenna a_j. Let $d(i, j)$ denote the distance between antenna a_i and antenna a_j. $PL[x]$ denotes the pathloss over a distance x. P_i denotes the transmit power of antenna a_i which we have to calculate. The convention followed in the following equations is that antenna a_i is transmitting to its corresponding receiver $a_{i'}$ which forms a link, and antenna a_j is any other antenna that also happens to be transmitting at the same instant.

In equation 17, P_{min} denotes the minimum required power level to work above the ambient noise level. P_{min} has a value of about $-85dBm$ for 11Mbps reception, for commercial 802.11b receivers [3]. Note that equations 17, 18, and 19 are not in the dB space but in physical terms. This equation sets the lower bound for the minimum power required by any antenna a_i.

$$\frac{P_i \times G_{i,i'}}{PL[d(i, i')]} \geq P_{min} \tag{17}$$

In equation 18, j refers to all antennas a_j, such that a_j is transmitting at the same time as $a_{i'}$. As each term is in physical space, the interference is summed, which we were not able to do when we were in the dB space.

$$\text{Interf}(i) = \sum_j \frac{P_j \times g(j, i)}{PL[d(j, i)]} \tag{18}$$

Equation 19 signifies the constraints on transmit power of each antenna. Power received by $a_{i'}$ should be greater than the interference seen at $a_{i'}$ by a factor of SIR_{reqd} which is the minimum Signal to Interference ratio required for meaningful transmission.

$$\frac{P_i \times G_{i,i'}}{PL[d(i, i')]} \geq SIR_{reqd} \times \text{Interf}(i') \tag{19}$$

Once again we have a set of linear equations, that we can use to solve for the transmit power of various antennas.

5 Other considerations

5.1 Adjacent-channel Interference

It was decided not to implement adjacent-channel interference as our model is intended only to test topologies where all links communicate across the same channel. All other transmissions will take place in non-overlapping channels (there are 3 non-overlapping channels in IEEE802.11b), and we do not have to consider any interference caused by them. Figure 4 shows the channel overlap. We intend to have all transmissions along channels 1,6 and 11 which will not interfere with each other. Hence our model needs to take into consideration only one channel at a time.

5.2 Antenna Characteristics

We have designed our model to contain 3 different types of antennas:

- Parabolic Antenna

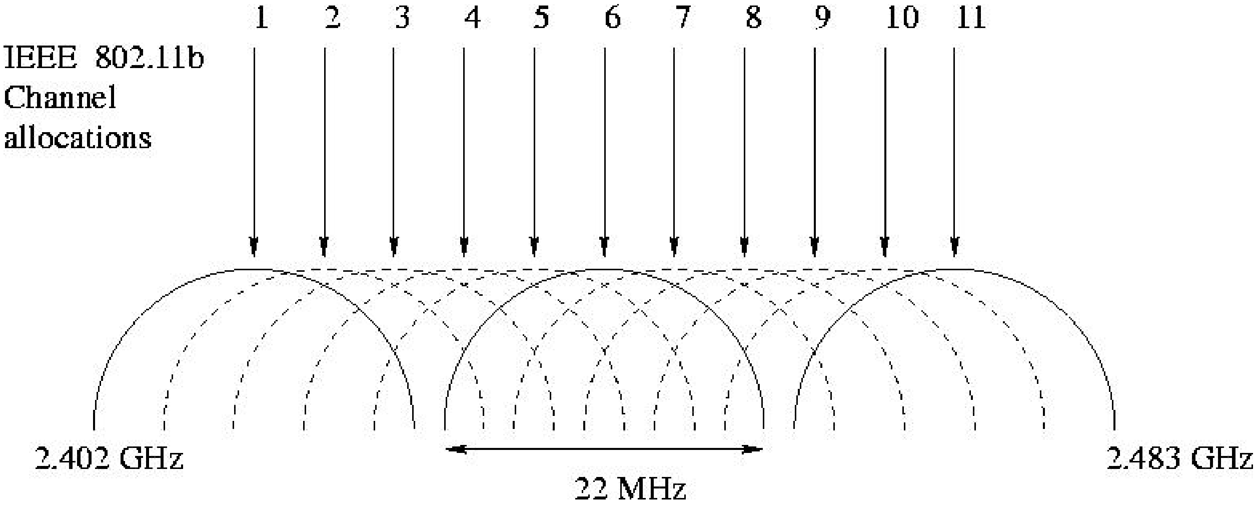


Figure 4: IEEE 802.11b Channel Specifications

- Patch Antenna
- Yagi Antenna

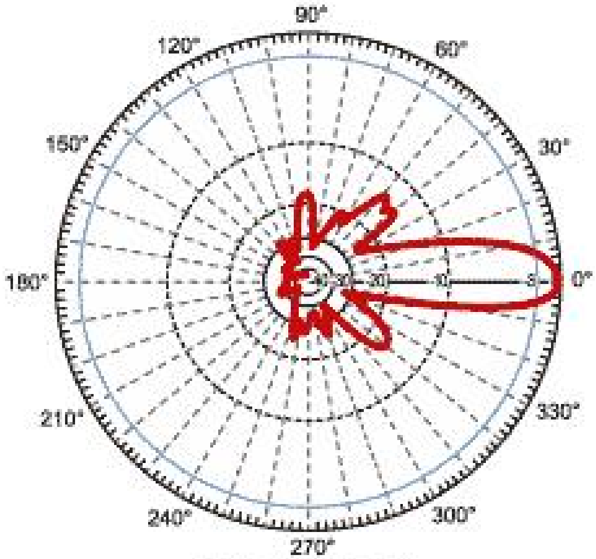
Figure 5 shows the vendor specifications of a 24dB High Gain Parabolic Antenna [4], which we modelled as shown in Table 2. If the angle between the original direction of the antenna and the other antenna under consideration is less than 10^o then the gain is $24dBm$. If the angle is between 10^o and 45^o then the gain is $-1dBm$ and similarly if the angle is greater than 90^o the gain is $-16dBm$.

Angle (Degrees)	**Gain (dB)**
< 10	24
10 – 45	-1
45 – 90	-6
> 90	-16

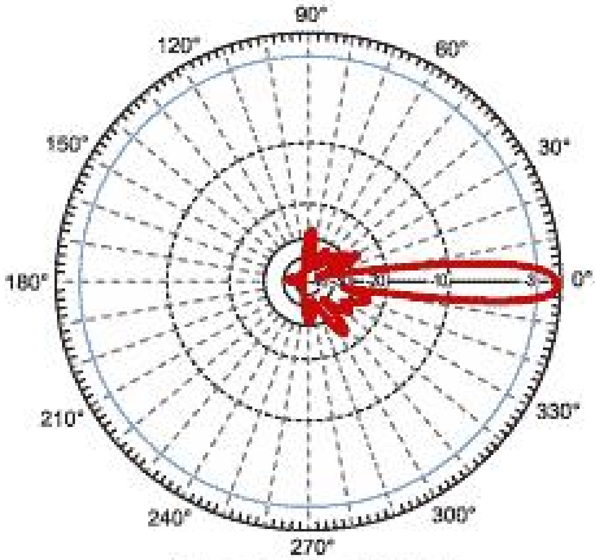
Table 2: Angle Difference and Corresponding Gain

5.3 The Z dimension

The various topologies created assumed a 2-dimensional space and hence, each antenna would have a fixed X and Y coordinate. We have added the functionality to allow users to specify the Z coordinate of antennas, hence creating more realistic topologies which will simulate altitude also. The distance between any 2 antennas depends on the Z coordinates also. Furthermore, the gain between two antennas for transmission and reception now vary in a 3 dimensional space. We have tackled this problem by first considering the angle difference along the X-Y plane and then considering the angle difference along the X-Z plane. We use vendor specifications as in section 5.2 for the same. Table 2 shows the Angle Difference and Gain value on the X-Y plane.



Vertical



Horizontal

Figure 5: Parabolic Antenna Specifications

5.4 Power Level Stability

The stability of the power level received at any given antenna can vary through the duration of long periods due to various factors such as atmospheric conditions. While this cannot be simulated, it is necessary for us to allocate power in such a way that the signal to interference ratio is not just achieved but rather maximized. This would ensure that any fluctuations in the signal level should still keep the signal to interference ratio above the required threshold for meaningful communication. Figure 6 shows the fluctuation in received power levels at various times[3]. The received power is in terms of percentage while the time indicates about 32,000 time steps during the day. The correlation between the power in terms of percentage and in terms of dBm depends on the card used for the experiments and this correlation was found out from [5]. The mean of the distribution

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[3] A CS625 project group of Satyam Sharma and Nityanand Rai conducted these experiments

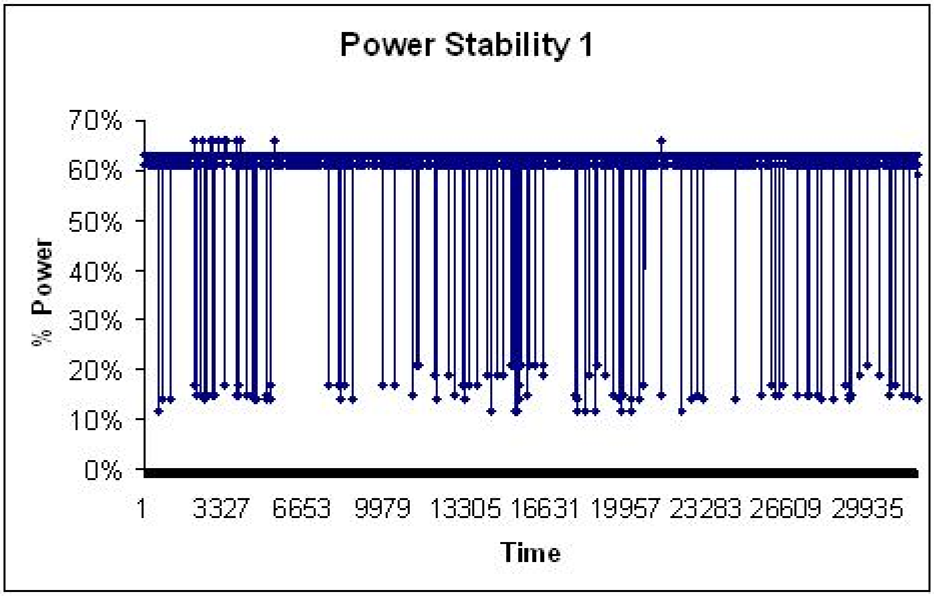

Figure 6: Power Stability Levels

in figure 6 is 63%, and the standard deviation is 2.83%. This corresponds to 95% of the distribution lying within a range of 5.66% which corresponds to approximately $4dB$ [5]. This implies that our power allocation should be able to handle a variance of about $2 - 4dB$ in the transmit power of any antenna. This is handled in 2 separate ways :

- The minimum receivable power for standard IEEE802.11b receivers is $-85dBm$ [3] but we can model the minimum receivable power to a higher value like $-75dBm$. This will ensure that even if the power level received drops a little, it is still well over the threshold of $-85dBm$.
- We can find the maximum achievable signal to interference ratio for a given topology, and a feasible power allocation for the minimum possible signal to interference ratio (which is a function of bitrate). We can now determine a signal to interference ratio between these two bounds that will be able to handle the fluctuations in the power stability levels. We calculate the maximum achievable signal to interference ratio in the results section.

5.5 Variable Bitrate

We have implemented the model such that different links in a topology can have different bandwidth speeds as this will affect the corresponding Signal to Interference ratio required for the receiving antenna. We have tried varying bandwidth on different links for given topologies but have noticed that this does not significantly change the power allocation of various antennas. Furthermore, we tried to change bandwidth on topologies which did not hold any feasible power allocation, and the

Bandwidth(MBPS)	**SIR required (dB)**
11	15
5	10
2	5

Table 3: Bandwidth and Corresponding SIR required

reducing of the bandwidth did not produce any significant change to alter the infeasible nature of the topologies.

QSopt

We used the QSopt [6] Linear Programming callable library and implemented this whole model in C++. We decided to shift from MATLAB when we tried to solve equations for larger topologies, but the results took an unreasonably long amount of time, probably due to some problem with the MATLAB server. The QSopt library has a wide range of functionality to define and modify various linear programming problems. The problem with modifying a set of equations is that if it is infeasible, it is hard to extract enough information to figure out which set of variables might need to have their parameters changed in order to make the set of equations feasible. An example of this is that we implement each antenna as a variable (or each antenna's transmit power) and then we generate the set of equations for the topology. If QSopt is not able to find a feasible solution for this set of equations, it is not able to give back enough relevant information for us to determine if we should let's say, reduce the bitrate of a certain link or antenna to sufficiently modify the set of equations in order to make it feasible.

Results

We have successfully allocated power to a district whose topology is shown in Figure 7 and consists of 254 antennas and spans 128 villages. In total, we have successfully allocated power to 761 different topologies, ranging from 32 to 128 villages. Various topologies were created in case of antennas at any particular village failing or going down, and feasible power allocations were found for these topologies too. This clearly shows the feasibility of inter-networking the district using 802.11b antennas.

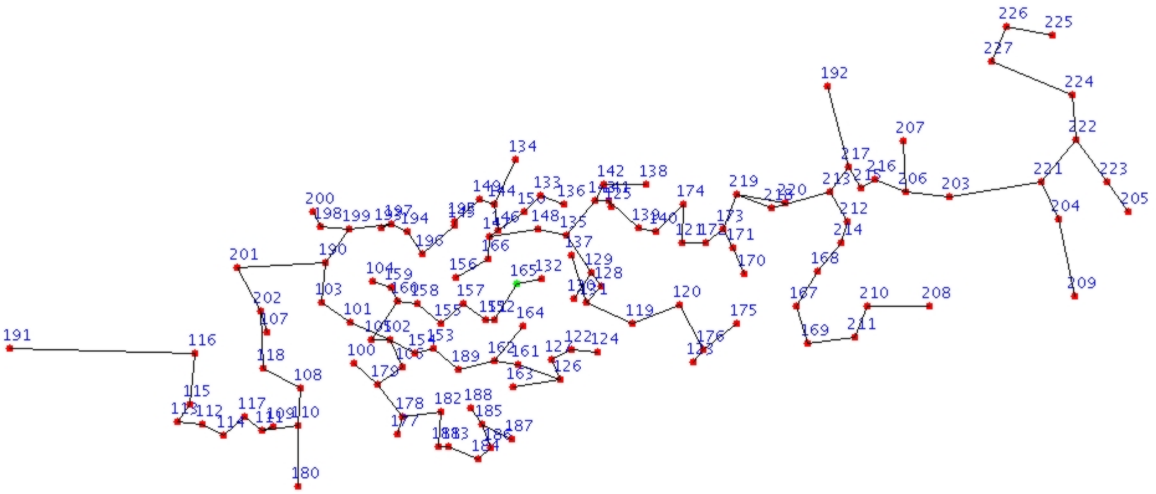

Figure 7: District Topology with 128 nodes

Some statistics of the various power allocations for the district are given below : A given district **D** of 128 villages, was divided into 4 quarters **Q1, Q2, Q3 and Q4** each consisting of 32 villages,

	Q1	**Q2**	**Q3**	**Q4**	**H1**	**H2**	**D**
Average SIR achievable (dBm)	23.015	23.904	23.968	24.126	23.062	23.125	21.941
Standard Deviation (dBm)	0.335	0.295	0.251	0.706	0.327	0.333	0.699

Table 4: District Topologies and achievable SIR

and then 2 halves **H1 and H2** each of 64 villages and various possible topologies were created for each of these divisions. Table 4 shows the cumulative results of the power allocation. For each quarter of the village, we used 63 different possible topologies. For each half we used 127 possible topologies, and for the entire district we used 255 different topologies. The reason for generating so many topologies is not research based but more of a practical nature. We intend on providing connectivity to various districts and should be able to take into consideration node failures at any given village, hence the various topologies to take care of such cases. The various topologies were taken into consideration to accomodate cases of link or node failures. The topology constructed is a tree and therefore for a 32 village district there are 31 seperate cases of node failures and of link failures. Along with the case where there is no failure, we arrive at 63 topologies for a 32 village district. From the table 4 we can see that we can allocate power to the entire district **D** such that the average achievable Signal to Interference ratio for any of the topologies is $21.941dB$. This would mean that all antennas will normally receive a signal to interference ratio much higher than the specified minimum of $10 - 13dB$ [2] for meaningful communication.

We are now in the position to simulate the power requirements of any given topology before actual deployment on the field. We have successfully determined the power requirements for various antennas of the district and can use this data once actual deployment starts.

Acknowledgments

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