Computation of Forward Kinematic model for Aldebaran Nao

A Transformation Matrix Approach using modified D-H parameters

s

Introduction

Nao is a popular humanoid robot with 25 Degrees-of-Freedom (DOF), manufactured by Aldebaran Robotics.it has been selected as the standard humanoid candidate for Robocup - the, now popular, Standard platform league(SPL). It is, also, being extensively used in diverse field of scientific study in premier technical institutions - range including but not limited to humanoid robotics, Vision problems, Text-to-Speech reconition, behaviour learning etc.It can work in interactive environment and recently been reported to be well-suited to work with autiistic children.More than 2,000 Nao exits in this earth, outside Aldebaran Robotics.We at IIT kanpur, usinng Nao for humanoidd robotics and robotic vision problems, are a small part of a significant global community.

Figure 1: *A view of Nao.* **NaoH25** – **ACADEMIC** model robot from Aldebaran

Figure 2: *Another view of Nao.* **NaoH25 – ACADEMIC**

Hardware Specification

Nao is equipped with a lot of hardware facilities and interfaces. It has Sonars for obstacle avoidance and distance measurement,two hi-definition video cameras, forse sensitive resistors(FSR) in foot to correctly determine pressure during active walking, LEDs to indicate booting process , tactile sensors, omnidirectional microphones and stereo broadcast system. The detailed hardware specification of Nao is a rather involved one and can be found in Nao Documentation by Aldebaran. We will only consider those hardware details which are more general and pertaining to our objective.Throughout this document, our analysis will refer to a particular version of Nao viz. **NaoH25 – ACADEMIC**. It is a 25 DOF humanoid robot with 5 DOFs in each arm; 6 DOFs in each of its legs; 2 in its Head and 1 Pelvic or hip joint near Torso. Nao does have three fingers which can open and close simultaneously but they are not considered as a possibility to add to the degrees of feeedom.

The outerbody of Nao is made up of light-weight technical plastic and Nao Weighs close to 5 kilograms.The 25 mechanical joints that Nao posseses can be electrically actuated by motors of two different types, mostly varying in rpm and current-torque rating.The stiffness of the joints can be varied externally by providing different motor torques.When an external torque, greater than the percentage of the maximum torque defined by the Stiffness, is applied to the joint, the joint will move in the direction of this external torque. Once this external torque disappears, the joint will go slowly to its last command angle.

The Inertial Unit of Nao is located within its chest with its own processor. The output data enables an estimation of the chest speed and attitude

(Yaw, Pitch, Roll). The central unit consists of two axis gyrometers and three accelerometers An Aldebaran Algorithm to compute the torso Angle from accelerometers and gyrometers is embeded inside the inertial board.The accelerometer is the only absolute reference and give good torso angle in static mode. When some motion is detected, the output angle is computed with gyrometers which have a good behavior in dynamic. However, integration of gyrometers creates a bias of the computed angle, so in dynamic mode, a fusion of compute angle from accelerometer and gyrometer is done to reduce this bias.

Objective/Approach

Within the present scope, our objective is:

:To compute the Forward Kinematic Model of Nao i.e. to develop a systematic mathematical analysis for Nao mvement when a combination of command angles are provided to Nao joint actuators so that the information about joint-space variables and the spatial geomemtry of the linkages allow us to know about operational space description of Nao and its end-effectors' description.The result can directly be applied to many problems related to Nao motion and vision .Also different segments of the results can be used to analyse a joint with multiple DOFs and any tree like robot structure with successve parallel or perpendicular axes.

:To Obtain a Model which can be used in training/ executing taught behaviours or work in a TEACH and PLAYBACK environment.

:To obtain a basis from where one can try to compute the inverse kiinematics for Articulated chains of Nao using algebraic methods.

:To compare the data obtained experimentally and programatically to that derived from mathematical analysis for further study or to understand the other mechanical artifacts and backlashes/ dead zones that Nao suffers with time and usage.

We will regidly affix a parental co-ordinate system $\mathbf{\Gamma}$ to Nao's inertial unit near torso and describe all positions and orientations concerning Nao with respect to this $\mathbf{\Gamma}$. We will also consider, theoritically, a universal co-ordinate system $\mathbf{\Upsilon}$ somwhere within the realms of the universe (may be, in the corner of the room in which our Nao moves or executes some behaviours) to which $\mathbf{\Gamma}$ can refer. Now the Humanoid nao can be visualised as a Robot Tree and each articulated chain can ve looked upon as a branch of one such tree. It is on those branches, independently, we will perform our analysis.This type of assumption recedes the complicacy of the problem at hand and allows us to reconsider the humanoid as a combination of articulated chains. We will consider Five such Chain viz. HeadChain, LeftArmChain, RightArmChain, LeftLegChain and RightLegChains.We will attach a frame, replica of $\mathbf{\Gamma}$ in terms of orientation but translated by a known distance, to each articulated chain , adjacent to the location where the each chain intersect the inertial unit structure. In effect, we will attach a co-ordinate system for nao left arm in the very vicinity of the left shoullder joint. The exact location of the origin of the frame will soon be determined.

We could have affixed $\mathbf{\Gamma}$ in some other location of Nao also ; e.g. in the support space of left leg and compute the forward kinematic model but, placing it near the Torso, in the most inertial unit of the structure, has certain advantages and also as the location, intuitively, is symmetricaly situated with respect to all articulated chains and the choice of the same offers distinct similarity between chain transformations.

So, we will have five replica of $\mathbf{\Gamma}$ across the whole-body-manipulator of Nao. Once we know the position of the end-effector frame in those replica frames we can easily compute the transformation for the end-effetor in $\mathbf{\Gamma}$-frame by per forming translational operations on the acquired results.These Translational operations are termed as offsets for joints and links. The numerical values of such offsets can be obtained by measurement. We will list a few of those, abstracted from Aldebaran list of Nao link lengths, below.

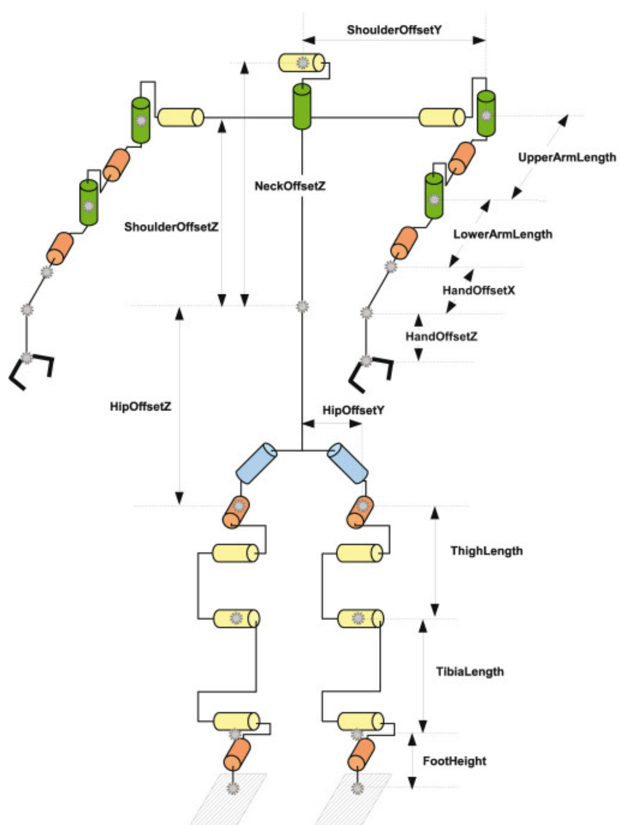

Figure 3: *Nao Kinematic Structure.* lengths are given below

Nao Body Part	length(in mm)
Upper Arm length	90.00
Lower Arm length	50.55
Thigh Length	100.00
Thigh Length	102.74
Sholder offset along $\mathbf{\Gamma_Z}$	100.00
Sholder offset along $\mathbf{\Gamma_Y}$	98.00
Hip offset along $\mathbf{\Gamma_Z}$	84.79
Hip offset along $\mathbf{\Gamma_Y}$	49.79
Hip offset along $\mathbf{\Gamma_X}$	58.00
Hip offset along $\mathbf{\Gamma_Z}$	12.31

Choice of D-H parameter and Summary of Frame Assignment

The celebrated Denavit and Hartenberg parameters simplifiy the spatial description of serial manipulators. The method uses the parameters that define the relation between two straignt lines in space.This method gives a rapid and precise way for computing the forward Kinematic model of the robot. In order to easily locate each link, we define a corresponding coordinate frame: frame i is rigidly attached to the link i. Denavit and Hartenberg proposed a matrix method to assign coordinate systems to each link in an articulated chain where the matrices are formed using the listed DH parameters.

We affix the frame using the folloiwing usual conventon:

1. We indicate all Z_is for the axis of rotation for each joints.

2. We find the common perpendicular betwwen Z_i and Z_{i+1}. The point at which this common perpendicular(the X_is) intersects Z_i determines the origin of i-th the frame. if Z_i and Z_{i+1} intersect then the X_i is chosen so as to be perpendicular to both Z_i and Z_{i+1}.for parallel Z_i and Z_{i+1} we can locate the origin anywhere along Z_i but it is conviniently chosen so that we can use the documented/easily measurable lengths of Nao body-parts.

3. The Y_i is chosen so as to form a dextral system.

But there are several variants of DH parameter which co-exist in Robotics literature in addition to the original DH notation (e.g. proximal variants and distal variants).In proximal variants (perhaps, the more popular version; mentioned in Craig etc.), the frames are attached at the beginning of the link whenever a choice presents itself. They are so chosen for the last frame that the θ is zero for a prismatic joint and d is zero for a rotary joint. Also link i is defined between joint i and joint i+1. Though proximal variant offers more notational clarity, often one has to produce one extra transformation in order to determine the end-effector as the distance between the last joint and end-effector does not appear in the DH table. Also, if we define link i between joint i-1 and joint i then in certain case, such as in the case of Nao, we can incorporate more information about a link in DH table. Like, in proximal variant the immobile baseframe is so chosen that it is aligned with frame 1 when the command angle is 0.We remove this convention and instead put a frame similar to $\mathbf{\Gamma}$ in the begining of the chain and list the initial angle between X axis of $\mathbf{\Gamma}$ and that of first frame as the θ for the first frame (for no external command angle). If the initial angle between X axis of $\mathbf{\Gamma}$ and that of first frame is -$-\pi/2$ and command angle is θ then the corresponding D-H parameter is $(-\pi/2+\theta)$. if the command angle is $\pi/4$ then the corresponding DH parameter will be $((-\pi/2+\pi/4)=-\pi/2$

For our purpose we we will use thw slightly altered version of the distal variant of DH notation. i.e though the link frame will be attached to the end of the link whenever an option exists the a and α will be measured from Z_{i-1} axis. The significance, in starting from Z_{i-1}, lies in the fact that we will start measuring the orientation of the first frame in the first joint with respect to the replica frame of $\mathbf{\Gamma}$ placed in the same joint at the begining of the chain.So there exists, therotically, a link of zero length between joint 0 and joint 1with joint 0 representing the origin of the replica of $\mathbf{\Gamma}$.We add this first row to every DH table of articulated chains. Eventually we still incorporate all the essense that the proximal variant would have offered and a little more. The link and joint parameters, used to analyse, Nao are defined as below: **link length a**: the offset distance between the Z_{i-1} and Z_i axes along the X_{i-1} axis;

link twist α: the angle from the Z_{i-1} axis to the Z_i axis about the X_{i-1} axis with anticlockwise+ve;

link offset d: the distance from the origin of frame X_{i-1} to the X_i axis along the Z_i axis;

joint angle θ: the angle between the X_{i-1} and X_i axis about the Z_i axis anticlockwise +ve.

Some of the joint of Nao have two or three degrees of freedom. In such cases,we analyse them as single degree of freedom joints separated by ZERO spatial distance. This offers quite a bit of freedom and simplifies the DH parameter table and the matrices. We have indicated the multiple degrees of freedom joint in the table above.

Chain Name	Joint list in order of consideration
HeadChain	HeadYaw,HeadPitch
LeftArmChain	(ShoulderPitch,ShoulderRoll),(ElbowYaw,ElbowRoll),WristYaw
LeftLegChain	(HipYawPitch, HipRoll, HipPitch), KneePitch, (AnklePitch, AnkleRoll)

Following the constraints listed above we see , for Nao Arm, any two consecutive joint axis either intersect each other or joints spatialy not isolated, rendering all a's ZERO. The origin of the relpica $\mathbf{\Gamma}$((i-1)th frame) , i-th frame and (i+1)th frame coincide within joint i+1, ShoulderRoll. The origins of frames i+2 and i+3 coincide inside the joint ElbowRoll i.e. the joint i+3. By convention, we choose origin of last frame at the distal end of the link inside i+4. We use the upper arm length and lower arm length (documented values) as d_3 and d_5. We placed the frames for the last joint inside it, at the distal end of the link,especially to use the lower arm length in determining the point within an end-effector. Calculation of θ and α are rather straightforward.The assignment for both arms is similar.For the head of Nao (there is only one two degrees of freedom joint) origin of all the frames coincide. So all a's and d's are zero.

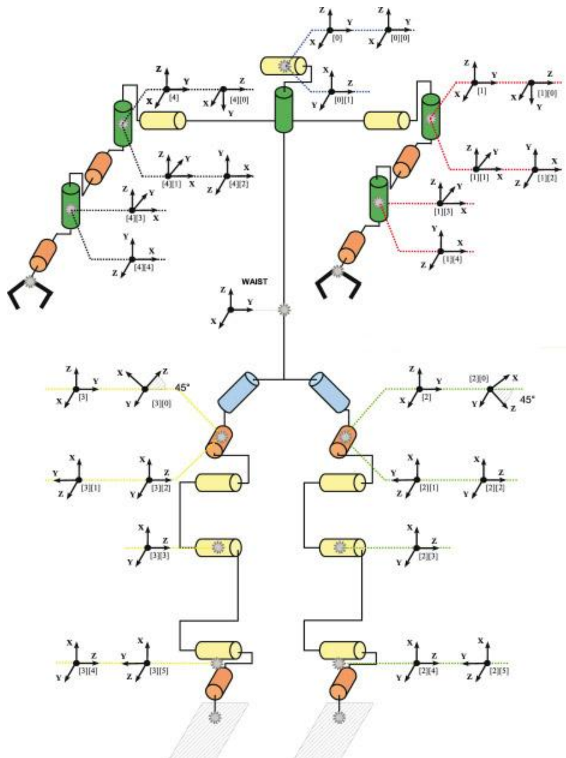

Figure 4: *Frame assignment for Nao Kinematics.* Joint Names and order of consideration is listed below

Similarly for Nao leg parallel pitch joints and other multiple degree of freedom joints render all d's ZERO.All the 3 origins of the frames pertaining to the 3-DOF(relpica $\mathbf{\Gamma}$,i, i+1, i+2) joint coincide inside the i+1 joint. Each Nao leg has a special joint HipYawPitch as the first joint of a 3 DOF joint near hip. The joint is so mounted that the Z axis of this joint is at $\pi/4$ with the horizontal $Y_{\mathbf{\Gamma}}$.But both the HipYawPitch joints of left and right leg can not be controlled independently as they are actuated by a single motor. Their Z-axis are perpendicular to each other. We find that α for this joint, in the left leg ,is -$3\pi/4$ as the Z-axis needs to be rotated by $3\pi/4$ clockwise to allign it to $Z_{\mathbf{\Gamma}}$. Hence α for this zoint in the right leg is $\pi/2$ - $3\pi/4 = -\pi/4$. Another difference between the left leg and right leg in terms of the DH parameter is that θ_{2L} is $\pi/4$ whereas θ_{2R} is -$\pi/4$ (for zero command angle). As the frame orientations for leftHipYawPitch and rightHipYawPitch are not same we expect the alignment of X-axis with HipRoll joint(which are symmetric and similar to each other) to be different. Rest of the parameters for both legs are identical.α for fourth and fifth joint are ZERO for each leg as Z-axis of third fourth and fifth joints are parallel(all pitch).We use the documented thigh length and tibia length as a_4 and a_5 but it is along the -ve X_{i-1}. So they come with a minus sign.

We list the DH parameter table, for each articulated chain below.

Nao Left Arm

i	α	a	d	θ
1	$-\pi/2$	0	0	θ_1
2	$\pi/2$	0	0	$\pi/2 + \theta_2$
3	$\pi/2$	0	UAL(90)	θ_3
4	$-\pi/2$	0	0	θ_4
5	$\pi/2$	0	LAL(50.55)	θ_5

Nao Right Arm

i	α	a	d	θ
1	$-\pi/2$	0	0	θ_1
2	$\pi/2$	0	0	$\pi/2 + \theta_2$
3	$\pi/2$	0	UAL(90)	θ_3
4	$-\pi/2$	0	0	θ_4
5	$\pi/2$	0	LAL(50.55)	θ_5

Nao Right Leg

i	α	a	d	θ
1	$-\pi/4$	0	0	$-\pi/2 + \theta_{1R}$
2	$-\pi/2$	0	0	$-\pi/4 + \theta_{2R}$
3	$\pi/2$	0	0	θ_{3R}
4	0	-Thigh length(-100)	0	θ_{4R}
5	0	-Tibia length(-102.74)	0	θ_{5R}
6	$-\pi/2$	0	0	θ_{6R}

Nao Left Leg

i	α	a	d	θ
1	$-3\pi/4$	0	0	$-\pi/2 + \theta_{1L}$
2	$-\pi/2$	0	0	$\pi/4 + \theta_{2L}$
3	$\pi/2$	0	0	θ_{3L}
4	0	-Thigh length(-100)	0	θ_{4L}
5	0	-Tibia length(-102.74)	0	θ_{5L}
6	$-\pi/2$	0	0	θ_{6L}

Nao Head

i	α	a	d	θ
1	0	0	0	θ_{1H}
2	$-\pi/2$	0	0	$-\pi/2+\theta_{2H}$

Transformation Matrices

The modified DH representation results in a 4x4 homogeneous transformation matrix for describing positional and orientational information between two successive frames .

$$\begin{bmatrix} cos\theta & sin\theta & 0 & a \\ sin\theta cos\alpha & cos\theta cos\alpha & -sin\alpha & -dsin\alpha \\ sin\theta sin\alpha & cos\theta sin\alpha & cos\alpha & dcos\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This base-to-end transformation matrix for each chain is obtained by multiplying the individual transformation matrices for succesive frame pairs. orders, in which joints are considered, are important. Despite the availability of all the Transformation matrices between consecutive joints we define two aditional type of Transformation matrices.The first of them retrieve each and every chain tranformation with respect to $\mathbf{\Gamma}$.We denote it as ${}_0\Gamma_{\mathbf{T}}$. this is only a translational transformation which takes the origin of each chain transformation from replica frame to original $\mathbf{\Gamma}$.This will be slightly different for each separate chains and will be listed in appropriate places. The other matrix that we want to bring in is denoted by ${}_n n'_{\mathbf{T}}$ where n is the number of the last joint/link starting from the begining of the chain. It is another known translational transform which when performed upon the end effector gives the location of the tool (might be a finger tip of Nao or a point on the lower surface of the footboard of Nao).The known hand and foot offsets are utilized for this purpose. Essentially, the last two matrices defined, do nothing in terms of orientational description but translates the derived co-ordinates from chain linkages to the $\mathbf{\Gamma}$ frame or to a point near the boundary of Nao's structure, at the end of the chain linkages.

As we already mentioned The Transformation betwween successive co-ordinate frames can be summarised in terms of a 4×4 T-Matrice viz. ${}_i i-1_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & sin\theta & 0 & a \\ sin\theta cos\alpha & cos\theta cos\alpha & -sin\alpha & -dsin\alpha \\ sin\theta sin\alpha & cos\theta sin\alpha & cos\alpha & dcos\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the table of DH parameter we find that for the first joint**HeadYaw**;- a,α,d are ZERO.So we derive the Transformation matrix between the

${}_1 0_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta cos0 & cos\theta cos0 & -sin0 & 0 \\ sin\theta sin0 & cos\theta sin0 & cos0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From the table of DH parameter we find that for the second and last joint in the*HeadChain* is **HeadPitch** a,α,d are ZERO.So we derive the Transformation matrix between the ${}_2 1_{\mathbf{T}}$=

$$\begin{bmatrix} cos(-\pi/2+\theta) & -sin(-\pi/2+\theta) & 0 & 0 \\ sin(-\pi/2+\theta)cos(-\pi/2) & cos(-\pi/2+\theta)cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin(-\pi/2+\theta)sin(-\pi/2) & cos(-\pi/2+\theta)sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics for Nao Arm

We will, primarily, consider the left arm for modelling. The analysis for right arm is very similar with a minor difference in Translation.

${}_1 0_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta cos(-\pi/2) & cos\theta cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin\theta sin(-\pi/2) & cos\theta sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_2 1_{\mathbf{T}}$=

$$\begin{bmatrix} cos(\pi/2+\theta) & -sin(\pi/2+\theta) & 0 & 0 \\ sin(\pi/2+\theta)cos(\pi/2) & cos(\pi/2+\theta)cos(\pi/2) & -sin(\pi/2) & 0 \\ sin(\pi/2+\theta)sin(\pi/2) & cos(\pi/2+\theta)sin(\pi/2) & cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_3 2_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta cos(\pi/2) & cos\theta cos(\pi/2) & -sin(\pi/2) & -sin(\pi/2)*90 \\ sin\theta sin(\pi/2) & cos\theta sin(\pi/2) & cos(\pi/2) & cos(\pi/2)*90 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_4 3_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta cos(-\pi/2) & cos\theta cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin\theta sin(-\pi/2) & cos\theta sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_5 4_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ sin\theta cos(\pi/2) & cos\theta cos(\pi/2) & -sin(\pi/2) & -sin(\pi/2)*50.55 \\ sin\theta sin(\pi/2) & cos\theta sin(\pi/2) & cos(\pi/2) & cos(\pi/2)*50.55 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Simplifying the matrices we find the following expressions for respective transformations ${}_{1}0_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin\theta & -cos\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_{2}1_{\mathbf{T}}$=

$$\begin{bmatrix} sin\theta & -cos\theta & 0 & 0 \\ 0 & 0 & -1 & 0 \\ cos\theta & -sin\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_{3}2_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ 0 & 0 & -1 & 90.00 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_{4}3_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -sin\theta & -cos\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_{5}4_{\mathbf{T}}$=

$$\begin{bmatrix} cos\theta & -sin\theta & 0 & 0 \\ 0 & 0 & -1 & -50.55 \\ sin\theta & cos\theta & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We will like to compute the mid point of the end effector for a fairly simple case with all configuration space values being 0 degree.

The transformation expression results in ${}_{0}5_{\mathbf{T}} = {}_{0}1_{\mathbf{T}} \times {}_{1}2_{\mathbf{T}} \times {}_{2}3_{\mathbf{T}} \times {}_{3}4_{\mathbf{T}} \times {}_{4}5_{\mathbf{T}}$. So ${}_{0}5_{\mathbf{T}}$ =

$$\begin{bmatrix} 0 & 0 & 1 & 140.55 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, to check the correcness of frame alignment we isolate The 3×3 rotation matrix ${}_{5}0_{\mathbf{R}}$ as

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Checking the orientation and comparing with the frame assignment we find that we had assumed proper affixation and transformation.beause we find $X_5.Y_0$ $=Y_5.Z_0=Z_5.X_0$ =1, meaning all the direction cosines are 0 degree, as it should be.

We also use the defined transforms, in order to locate the [x,y,z] in inertial Torso frame, 1. $_{\mathbf{\Gamma}}0_{\mathbf{T}}$=

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 98 \\ 0 & 0 & 1 & 100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Incorporate the hand offset to determine the tool we define $_55'_{\mathbf{R}}$=

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 58 \\ 0 & 1 & 0 & -12.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We have used the previously listed offset and length values in mm.

The Final co-ordinate for the mid point of the end-effector will be [140.55, 98, 100] in torso frame and $_5\mathbf{\Gamma}_{\mathbf{T}}$ will look like

$$\begin{bmatrix} 1 & 0 & 0 & 140.55 \\ 0 & 1 & 0 & 98 \\ 0 & 0 & 1 & 100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now the Inverse of the matrix gives us the posion and orientation of Γ in the end effector frame.We find $_{\mathbf{\Gamma}}5_{\mathbf{T}}$=

$$\begin{bmatrix} 1 & 0 & 0 & 98 \\ 0 & 1 & 0 & -100 \\ 0 & 0 & 1 & -140.55 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which again coforms to our general frame assignment. Indeed we need to move 140.55 mm along the negative WristYaw axis (Z of the last frame), 98 mm positive X axis of last frame and and 100 mm about Y axis to reach the origin of the Torso frame.

The Forward kinematic Analysis of Nao right arm perfectly mirrors the previous analysis and Hence the D-H parameters and the resulting transformation matrice expressions are identical for $_54_{\mathbf{T}}$ .But The Base Co-Ordinate Frame is situated at the right shoulder joint. So the only difference in the complete expression for the forward model for Right Arm will be the $_0\Gamma_{\mathbf{T}}$ for each arm.

Forward Kinematics for Left Leg of Nao

$_10_{\mathbf{T_L}}$=

$$\begin{bmatrix} cos(-\pi/2+\theta_{1L}) & -sin(-\pi/2+\theta_{1L}) & 0 & 0 \\ sin(-\pi/2+\theta_{1L})cos(-3\pi/4) & cos(-\pi/2+\theta_{1L})cos(-3\pi/4) & -sin(-3\pi/4) & 0 \\ sin(-\pi/2+\theta_{1L})sin(-3\pi/4) & cos(-\pi/2+\theta_{1L})sin(-3\pi/4) & cos(-3\pi/4) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_2 1_{\mathbf{T_L}} =$

$$\begin{bmatrix} cos(\pi/4+\theta_{2L}) & -sin(\pi/4+\theta_{2L}) & 0 & 0 \\ sin(\pi/4+\theta_{2L})cos(-\pi/2) & cos(\pi/4+\theta_{2L})cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin(\pi/4+\theta_{2L})sin(-\pi/2) & cos(\pi/4+\theta_{2L}))sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_3 2_{\mathbf{T_L}} =$

$$\begin{bmatrix} cos\theta_{3L} & -sin\theta_{3L} & 0 & 0 \\ sin\theta_{3L}cos(\pi/2) & cos\theta_{3L}cos(\pi/2) & -sin(\pi/2) & 0 \\ sin\theta_{3L}sin(\pi/2) & cos\theta_{3L}sin(\pi/2) & cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_4 3_{\mathbf{T_L}} =$

$$\begin{bmatrix} cos\theta_{4L} & -sin\theta_{4L} & 0 & -100 \\ sin\theta_{4L}cos(0) & cos\theta_{4L}cos(0) & -sin(0) & 0 \\ sin\theta_{4L}sin(0) & cos\theta_{4L}sin(0) & cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_5 4_{\mathbf{T_L}} =$

$$\begin{bmatrix} cos\theta_{5L} & -sin\theta_{5L} & 0 & -102.74 \\ sin\theta_{5L}cos(0) & cos\theta_{5L}cos(0) & -sin(0) & 0 \\ sin\theta_{5L}sin(0) & cos\theta_{5L}sin(0) & cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_6 5_{\mathbf{T_L}} =$

$$\begin{bmatrix} cos\theta_{6L} & -sin\theta_{6L} & 0 & 0 \\ sin\theta_{6L}cos(-\pi/2) & cos\theta_{6L}cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin\theta_{6L}sin(-\pi/2) & cos\theta_{6L}sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We will like to compute the mid point of the FootBoard for a fairly simple case with all configuration space values being 0 degree.

The transformation expression results in ${}_0 6_{\mathbf{T}} = {}_0 1_{\mathbf{T}} \times {}_1 2_{\mathbf{T}} \times {}_2 3_{\mathbf{T}} \times {}_3 4_{\mathbf{T}} \times {}_4 5_{\mathbf{T}} \times {}_5 6_{\mathbf{T}}$. So ${}_0 6_{\mathbf{T}} =$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -202.74 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, to check the correcness of frame alignment we isolate The 3×3 rotation matrix ${}_6 0_{\mathbf{R}}$ as

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Checking the orientation and comparing with the frame assignment we find that we had assumed proper affixation and transformation.beause we find $X_6.Z_0$ = 1, $Y_6.Y_0$=-1, $Z_6.X_0$ =1, meaning all the direction cosines are 0 degree, as it should be.

We also use the defined transforms, in order to locate the [x,y,z] in inertial Torso frame, 1. $_{\Gamma}0_{\mathbf{T}}$=

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 49.79 \\ 0 & 0 & 1 & -84.79 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

We have used the previously listed offset and length values in mm.

The Final co-ordinate for the mid point of the end-effector will be [0, 49.79, -284.79] in torso frame and $_{6}\boldsymbol{\Gamma}_{\mathbf{T}}$ will look like

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 49.79 \\ 1 & 0 & 0 & -287.53 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now the Inverse of the matrix gives us the posion and orientation of Γ in the end effector frame.We find $_{\Gamma}5_{\mathbf{T}}$=

$$\begin{bmatrix} 0 & 0 & 1 & 287.53 \\ 0 & -1 & 0 & -49.79 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

which agian coforms to our general frame assignment. Indeed we need to move Zero mm along the AnkleRoll(Z of the last frame), 284.79 mm positive X axis of last frame and and 49.79 mm about - Y axis to reach the origin of the Torso frame.

Forward Kinematics for Right leg of Nao

$_{1}0_{\mathbf{T_R}}$=

$$\begin{bmatrix} cos(-\pi/2+\theta_{1R}) & -sin(-\pi/2+\theta_{1R}) & 0 & 0 \\ sin(-\pi/2+\theta_{1R})cos(-\pi/4) & cos(-\pi/2+\theta_{1R})cos(-\pi/4) & -sin(-\pi/4) & 0 \\ sin(-\pi/2+\theta_{1R})sin(-\pi/4) & cos(-\pi/2+\theta_{1R})sin(-\pi/4) & cos(-\pi/4) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$_{2}1_{\mathbf{T_R}}$=

$$\begin{bmatrix} cos(-\pi/4+\theta_{2R}) & -sin(-\pi/4+\theta_{2R}) & 0 & 0 \\ sin(-\pi/4+\theta_{2R})cos(-\pi/2) & cos(-\pi/4+\theta_{2R})cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin(-\pi/4+\theta_{2R})sin(-\pi/2) & cos(-\pi/4+\theta_{2R}))sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$_3 2_{\mathbf{T_R}} =$

$$\begin{bmatrix} cos\theta_{3R} & -sin\theta_{3R} & 0 & 0 \\ sin\theta_{3R}cos(\pi/2) & cos\theta_{3R}cos(\pi/2) & -sin(\pi/2) & 0 \\ sin\theta_{3R}sin(\pi/2) & cos\theta_{3R}sin(\pi/2) & cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$_4 3_{\mathbf{T_R}} =$

$$\begin{bmatrix} cos\theta_{4R} & -sin\theta_{4R} & 0 & -100 \\ sin\theta_{4R}cos(0) & cos\theta_{4R}cos(0) & -sin(0) & 0 \\ sin\theta_{4R}sin(0) & cos\theta_{4R}sin(0) & cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$_5 4_{\mathbf{T_R}} =$

$$\begin{bmatrix} cos\theta_{5R} & -sin\theta_{5R} & 0 & -102.74 \\ sin\theta_{5R}cos(0) & cos\theta_{5R}cos(0) & -sin(0) & 0 \\ sin\theta_{5R}sin(0) & cos\theta_{5R}sin(0) & cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$_6 5_{\mathbf{T_R}} =$

$$\begin{bmatrix} cos\theta_{6R} & -sin\theta_{6R} & 0 & 0 \\ sin\theta_{6R}cos(-\pi/2) & cos\theta_{6R}cos(-\pi/2) & -sin(-\pi/2) & 0 \\ sin\theta_{6R}sin(-\pi/2) & cos\theta_{6R}sin(-\pi/2) & cos(-\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Forward kinematic Analysis of Nao right Leg differs for first two transformation matrices but the result perfectly mirrors the previous one except changes in sign. .But The Base Co-Ordinate Frame is situated at the right hipr joint. So the difference in the complete expression for the forward model for Right leg will be the $_0\Gamma_{\mathbf{T}}$ matrix.