Abstract:

A quasi-Gray code of dimension n and length ℓ over an alphabet Σ is a sequence of distinct words $w_1,w_2,\dots,w_\ell$ from Σ^n such that any two consecutive words differ in at most c coordinates, for some fixed constant $c>0$.

In this paper we are interested in the read and write complexity of quasi-Gray codes in the bit-probe model, where we measure the number of symbols read and written in order to transform any word w_i into its successor w_{i+1}.

We present construction of quasi-Gray codes of dimension n and length 3^n over the ternary alphabet $\{0,1,2\}$ with worst-case read complexity $O(\log n)$ and write complexity 2.

This generalizes to arbitrary odd-size alphabets. For the binary alphabet, we present quasi-Gray codes of dimension n and length at least $2^n - 20n$ with worst-case read complexity $6+\log n$ and write complexity 2.

Our results significantly improve on previously known constructions and for the odd-size alphabets we break the $\Omega(n)$ worst-case barrier for space-optimal (non-redundant) quasi-Gray codes with constant number of writes. We obtain our results via a novel application of algebraic tools together with the principles of catalytic computation. We also establish certain limits of our technique in the binary case.