CS350 Assignment 3

These are practice problems. Hints for solutions for problems 1, 2, 3 will be provided. You can submit solutions to the last two problems for extra credit, *on an individual basis.*

Some questions on the λ calculus.

1 . Apply β reductions to obtain the normal forms of the following expressions, if they are valid. Recall that the normal forms are those which will remain the same under β-reductions.

  a . $(\lambda x.xx)(\lambda x.y)x$
  b . $(\lambda x.xx)((\lambda x.y)x)$
  c . $(\lambda x.x(\lambda y.xyy)z)z$

2 . For the following λ-term, provide one sequence of β-reductions which yields us a normal form, and another which fails to terminate.

$$(\lambda xyz \cdot xz(yz))PMM,$$

where $P = (\lambda xy \cdot x)$ and $M = (\lambda x \cdot xx)$.

3 . Prove that the following is a fixed-point combinator - that is, for any λ-term f, we have $f\Theta f = \Theta f$.

$$\Theta = (\lambda x \lambda y.(y(xxy)))(\lambda x.\lambda y.(y(xxy)))$$

4 . (Extra Credit) Our definition of `nil` coincided with `False` and 0. However, our definition of `Tail` did not satisfy the criterion that `Tail nil = nil`. Give a representation of `nil` that satisfies this equation. Of course, it no longer would coincide with `False` and 0.

Show that your definition satisfies the constraint by deriving the equation by substitution. *Hint: What kind of property does the equation remind you of?*

5 . (Extra Credit) Give a definition of `nil` which satisfies `Head nil = Tail nil = nil`. Show that your definition satisfies this constraint by deriving the equation by substitution.