Stochastic Streams

Andrew McGregor
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IITK Data Streams Workshop 12/19/06

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a) Estimating properties of distributions.

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b) Processing non-deterministic data in streams.

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c) Where do streams come from?

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- Entropy: $\sum -p_i \log p_i$

$O(\epsilon^{-3} \log^5 m \log \delta^{-1})$
[Bhuvanagiri, Ganguly '06]

$O(\epsilon^{-2} \log m \log \delta^{-1})$
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- **Information Distances:** e.g. $\sum (\sqrt{p_i} - \sqrt{q_i})^2$

  Multiplicative Approx: All *f*-Divergences (except L_1) and Bregman-Divergences (except L_2) require $\Omega(n)$ space.

  Additive Approx: Bound *f*-Divergences, Jensen-Shannon...

  Embedding: Can embed Hellinger but not approximate
  [Guha, McGregor, Venkatasubramarian '06], [Guha, Indyk, McGregor '07]

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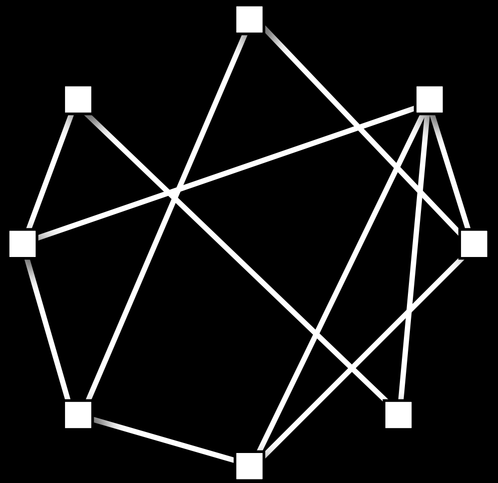

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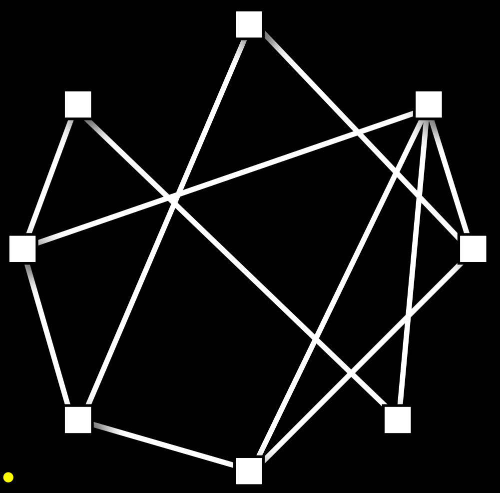

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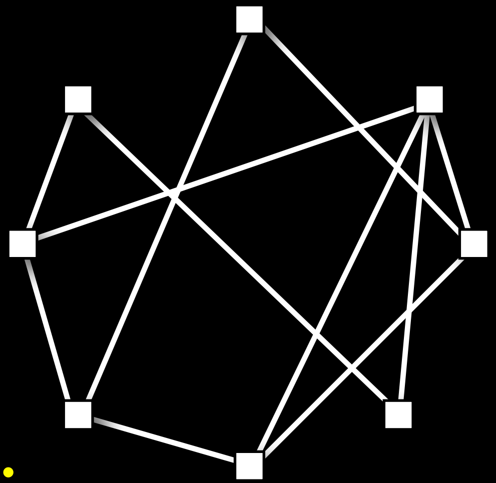

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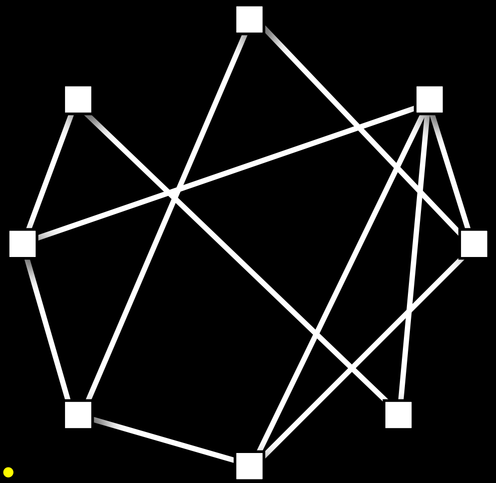

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- What about mixing-time, cover-time etc.?

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- Goal: Compute expected values of aggregates, e.g.

  Mean, Sum, F_1, Max [Jayram, Kale, Vee '07]

  Mean, Median, F_0, F_2 [McGregor, Muthukrishnan '07]

b) Processing non-deterministic data in streams.

- Thm: $O(\log n)$-pass $(1+\epsilon)$-approx for $E[Mean]$ in $O(\epsilon^{-1} \log n)$ space. [Jayram, Kale, Vee '07]

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   Then whp. $A=\{\mathit{Count}_j=E[\mathit{Count}_j] \pm O(\epsilon^{-1} \log n)\text{: for all } j\}$

   Maintain $\Pr[\mathit{Count}_j=z|A]$ & $E[\mathit{Mean}_j|A]$ in $O(\epsilon^{-1} \log n)$ space.

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- Who cares about the empirical distribution?!
- What if we don't know the probabilistic stream?!

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2. Quantiles
3. Learning Distributions

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- E.g. each A_i is identically distributed...

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  [Greenwald, Khanna '01], [Cormode, Korn, Muthukrishnan, Srivastava '06]

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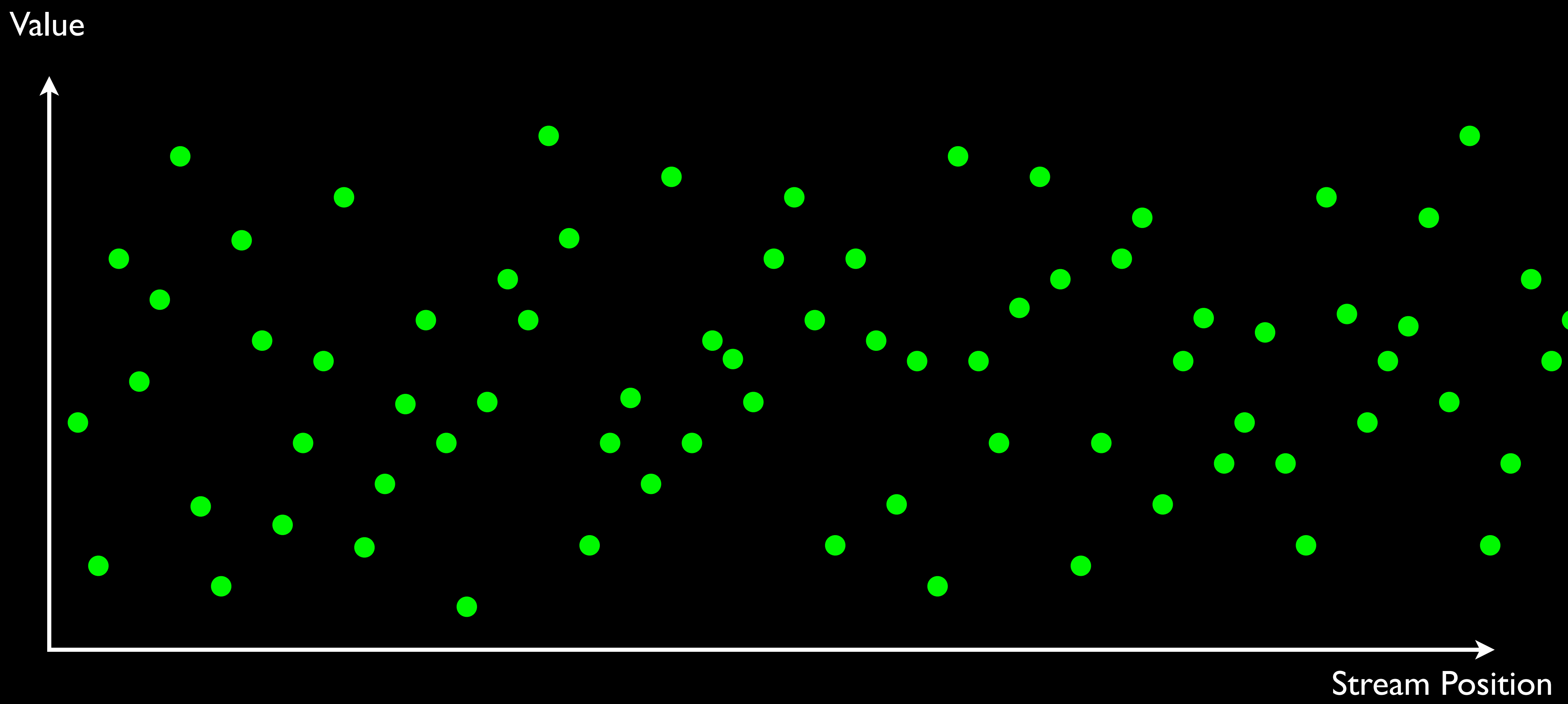
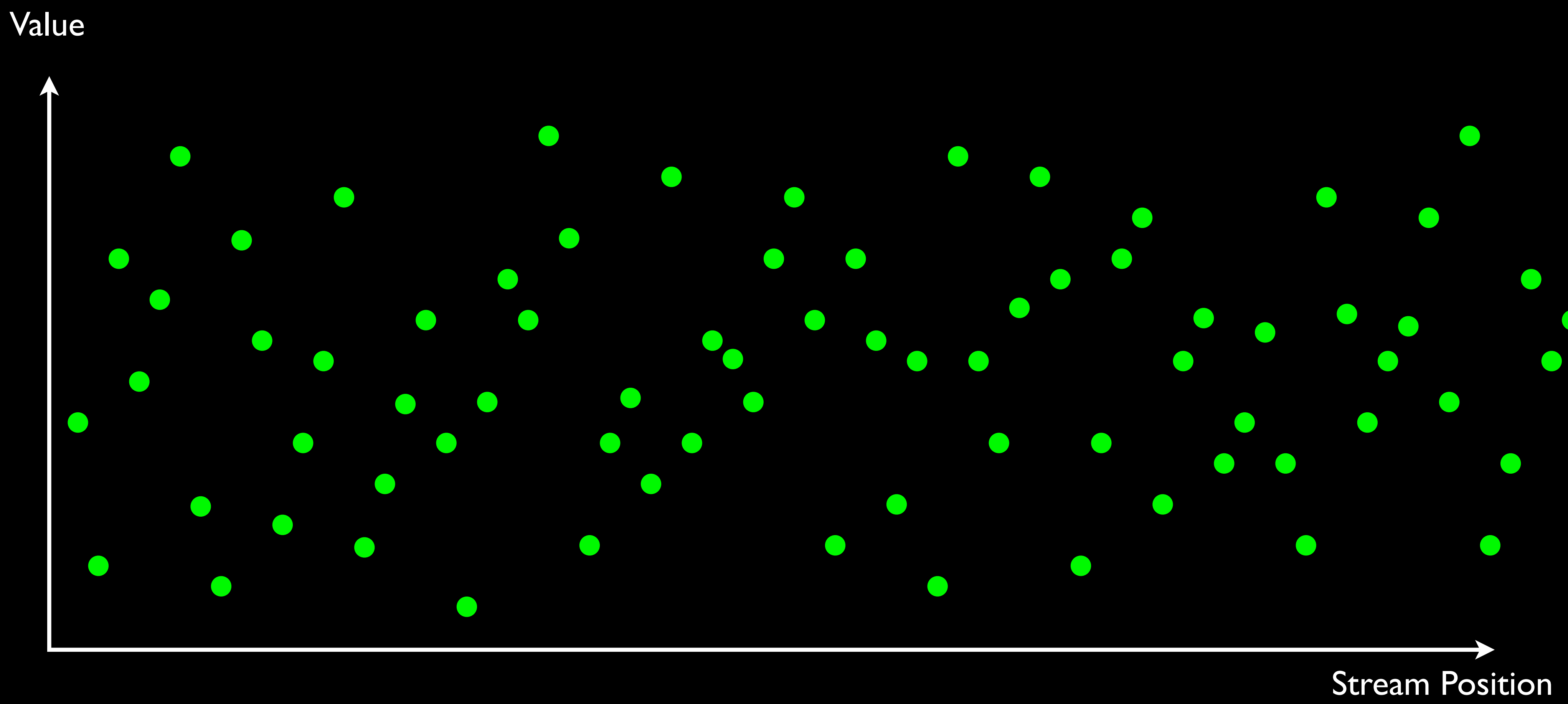
- Thm: Can find $O(m^{-1/2}\log m)$-approx median in O(1) words of space


Value
Stream Position

Algorithm: Maintain lower/upper bound [a, b] for median and c in [a,b]



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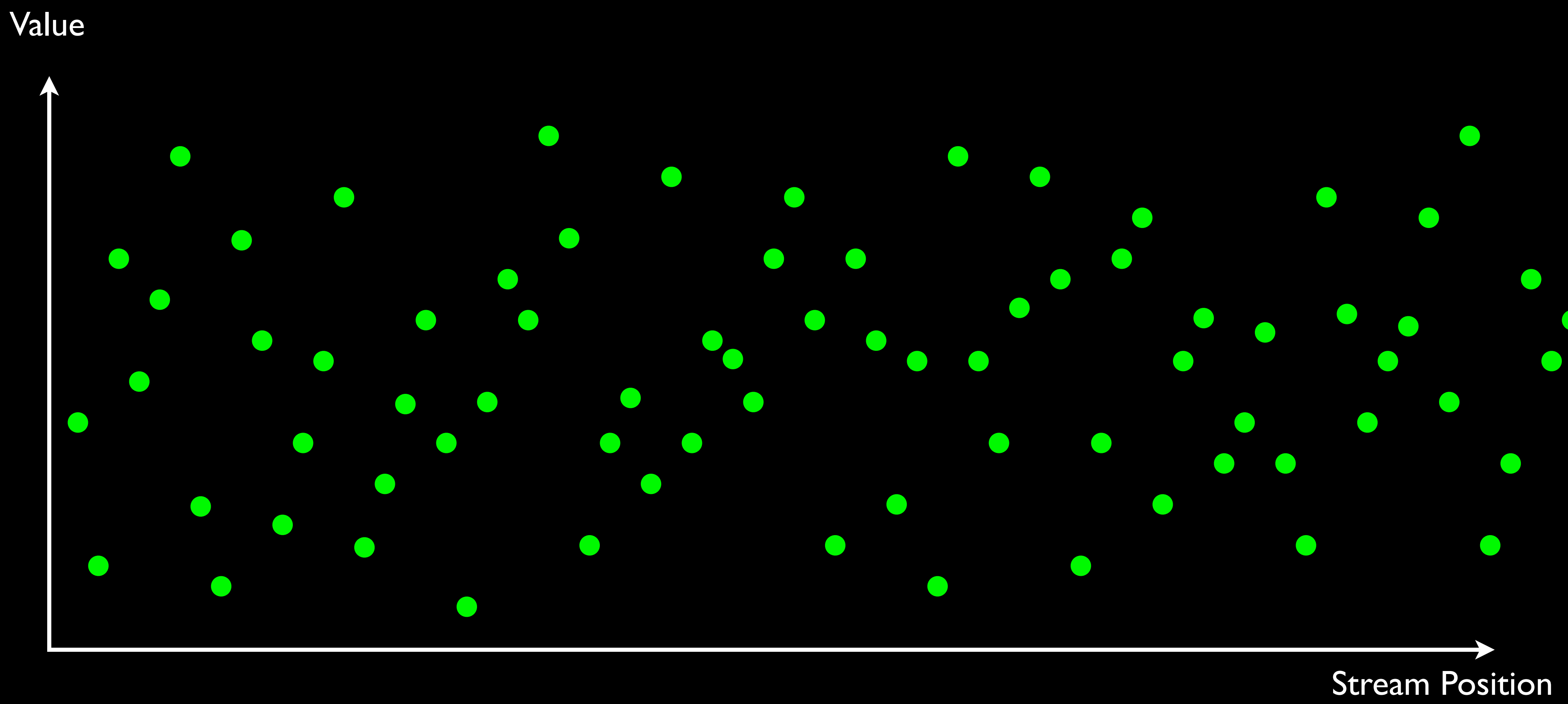

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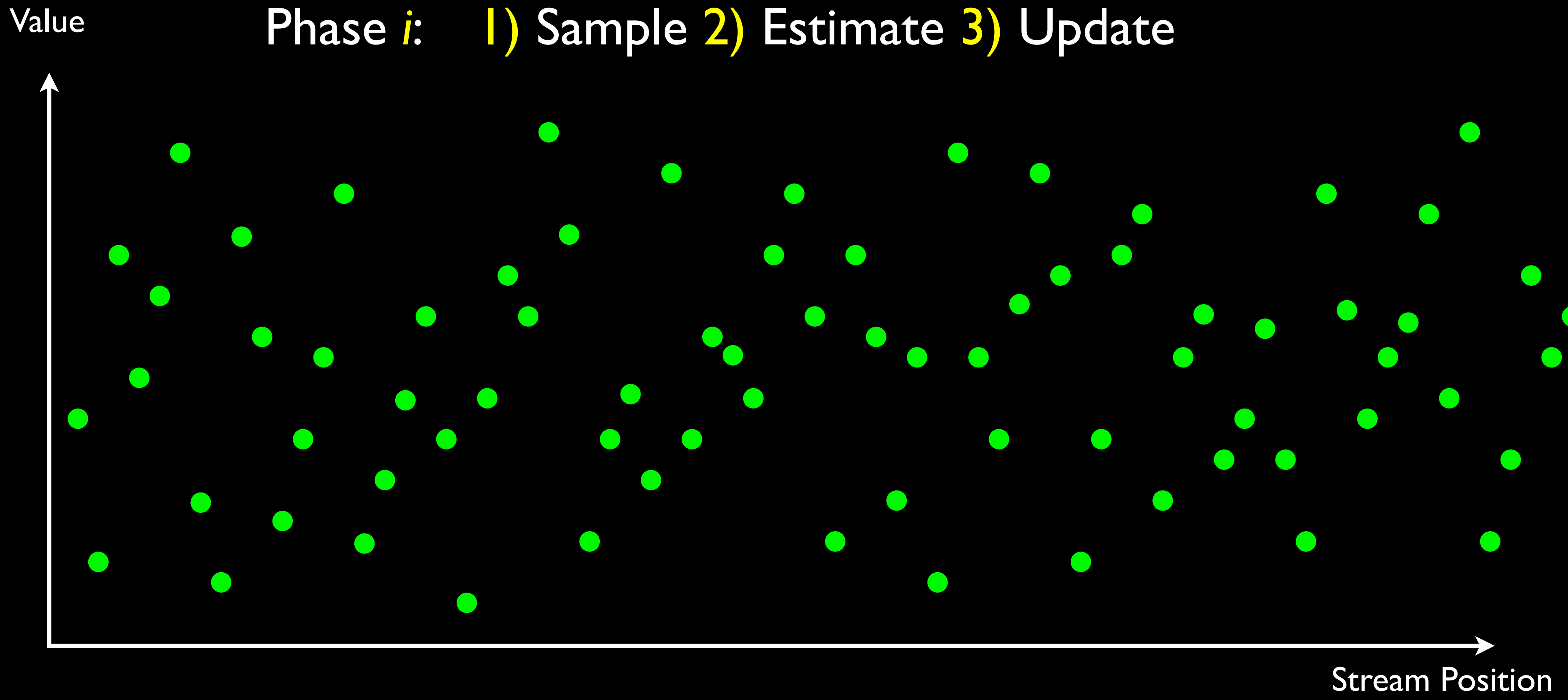

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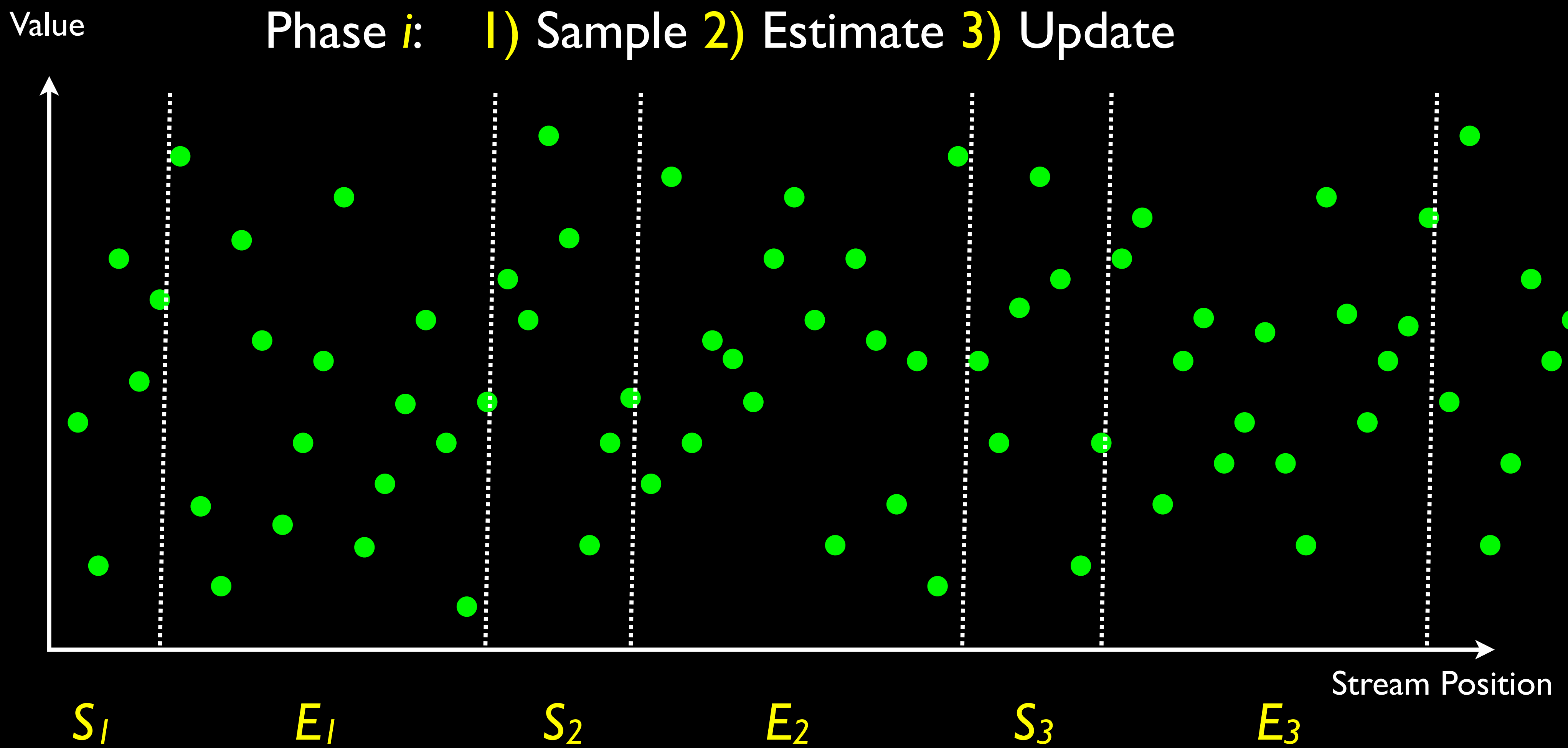

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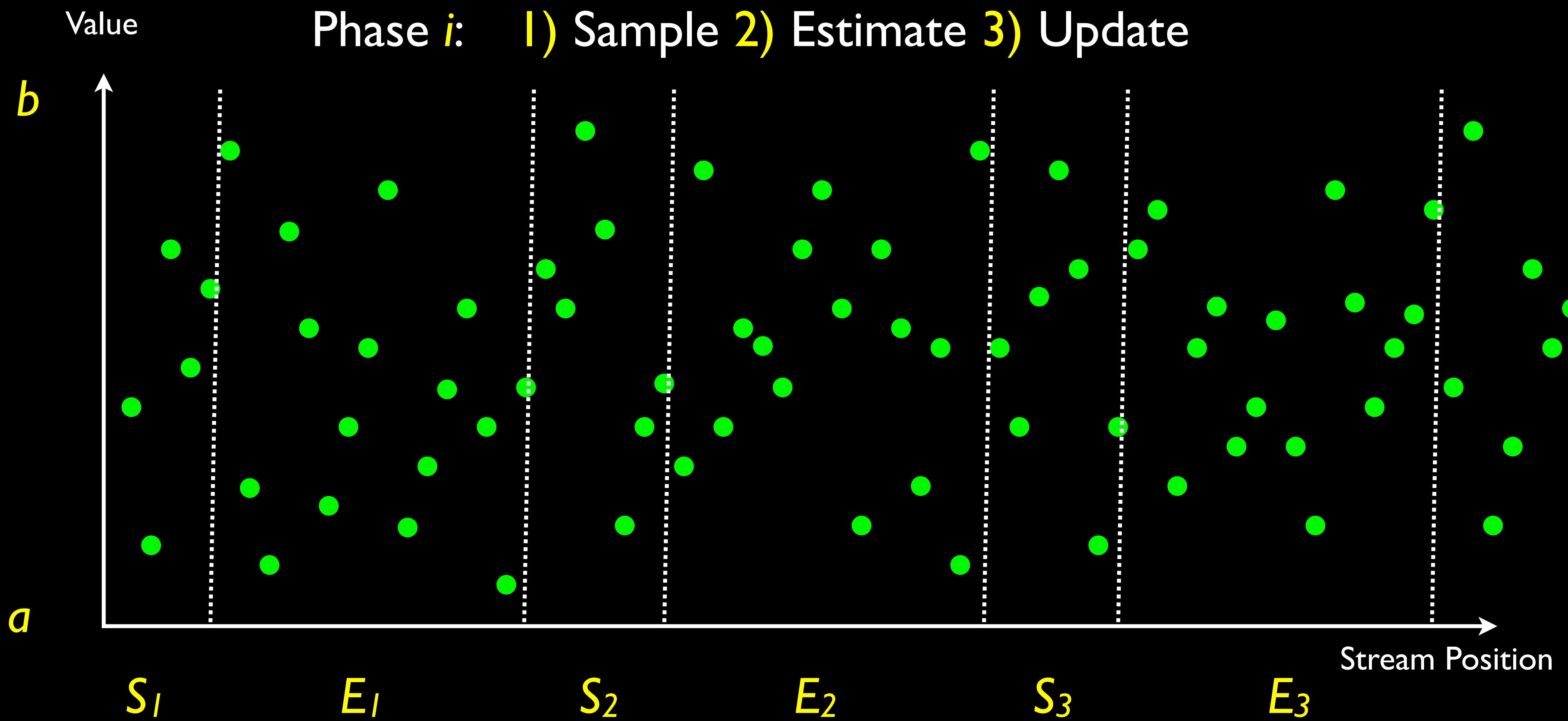

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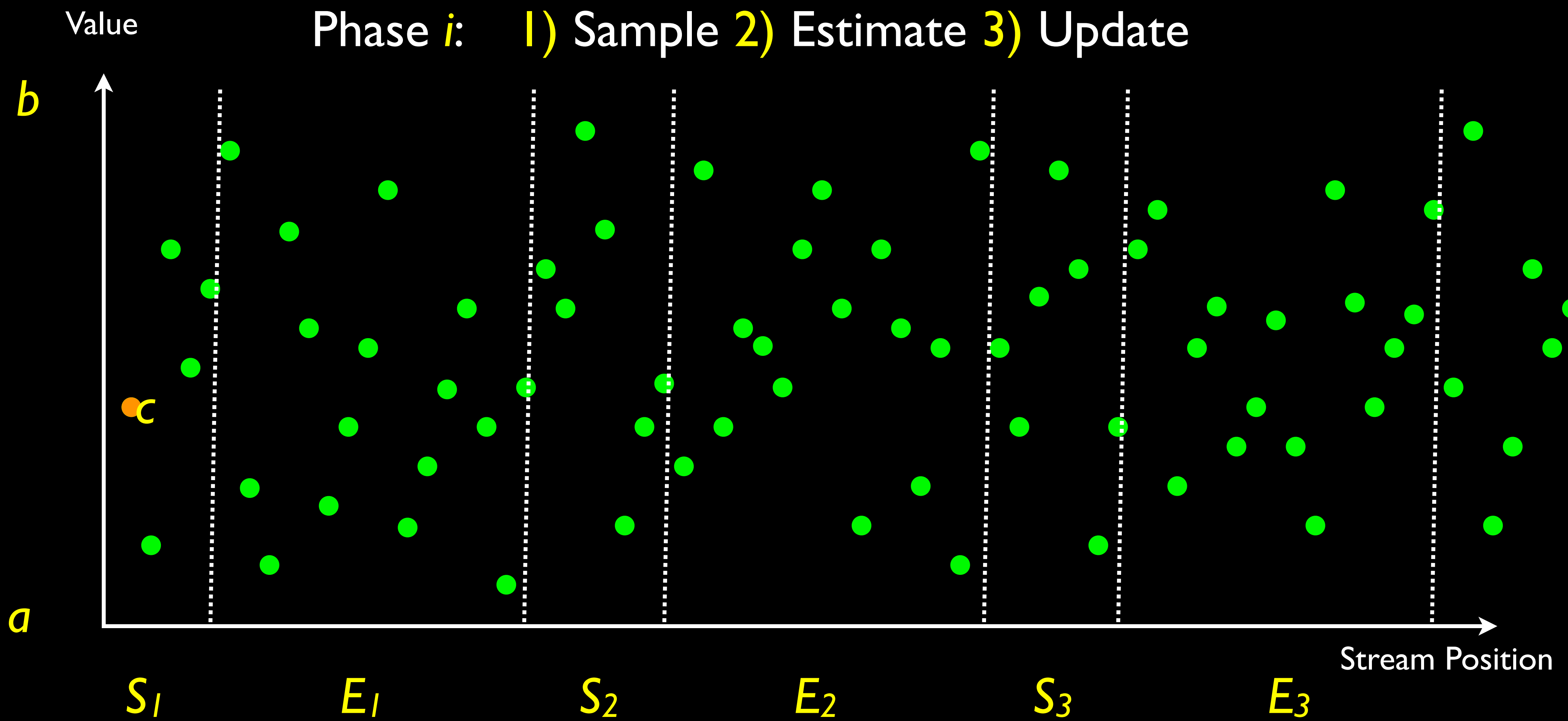

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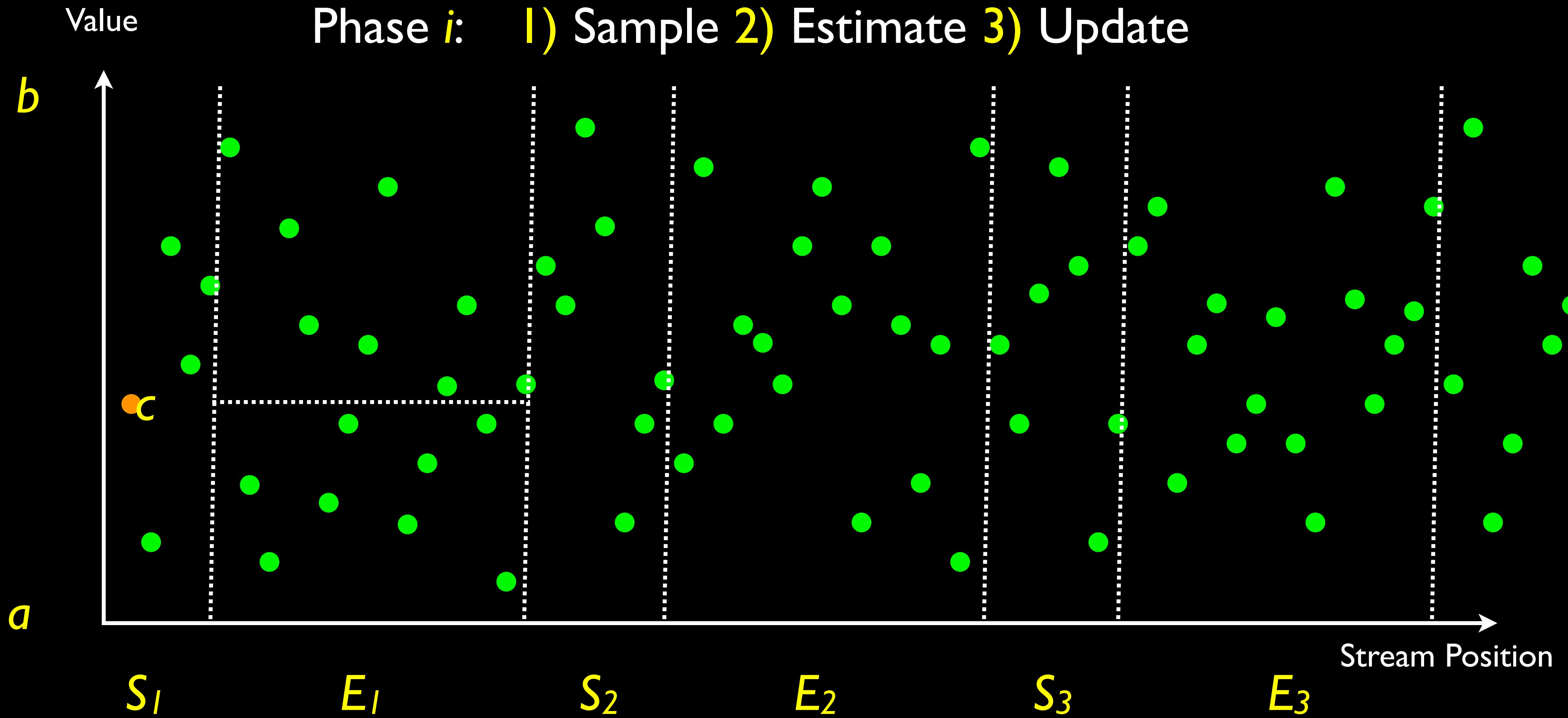

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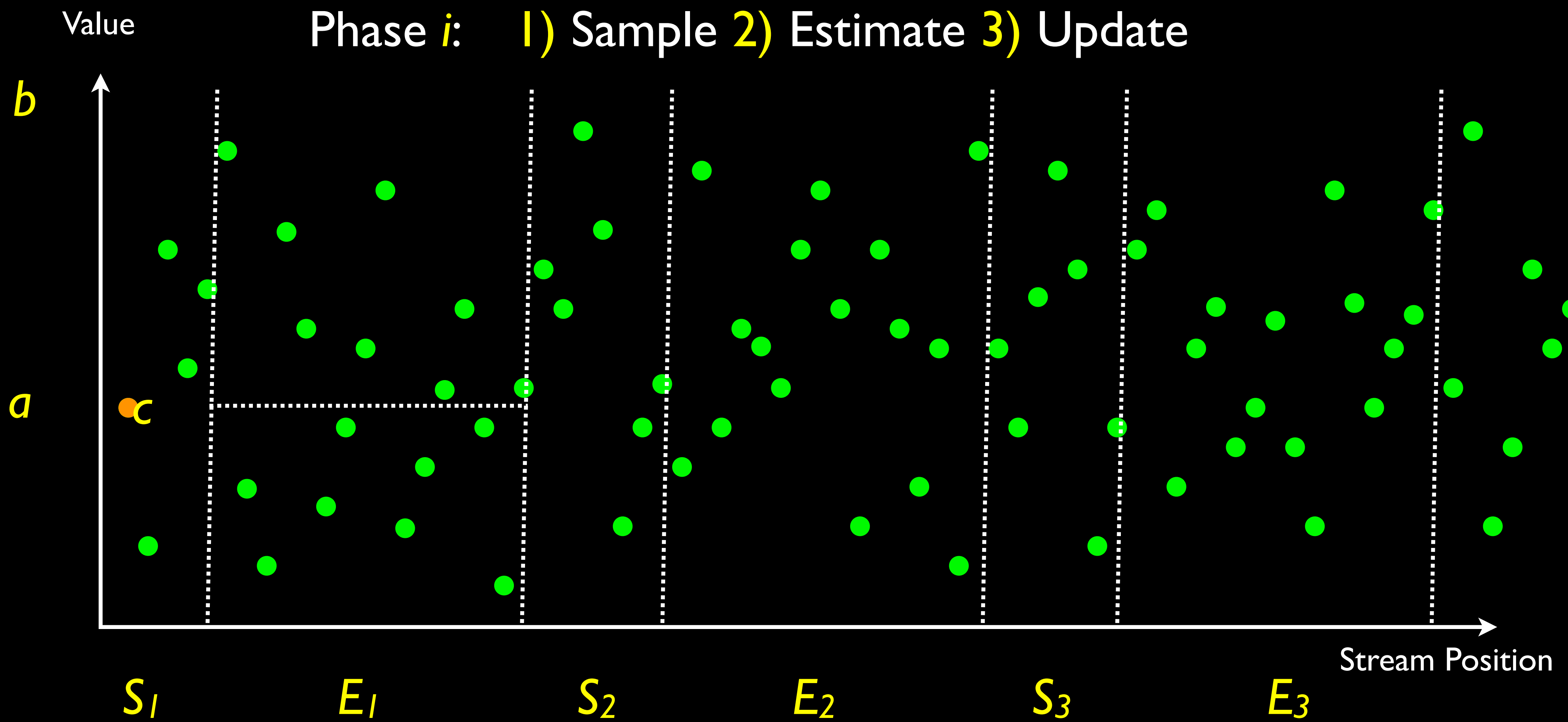

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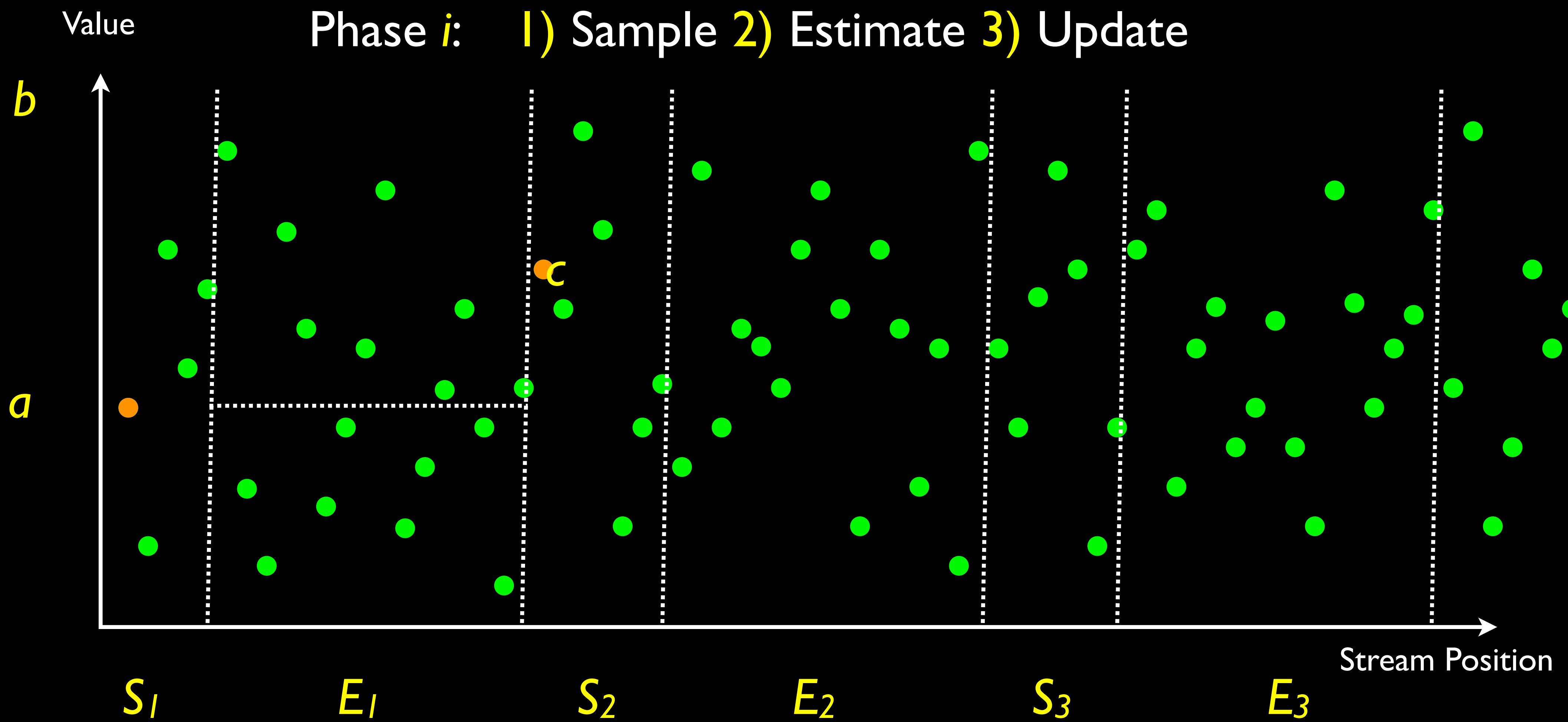

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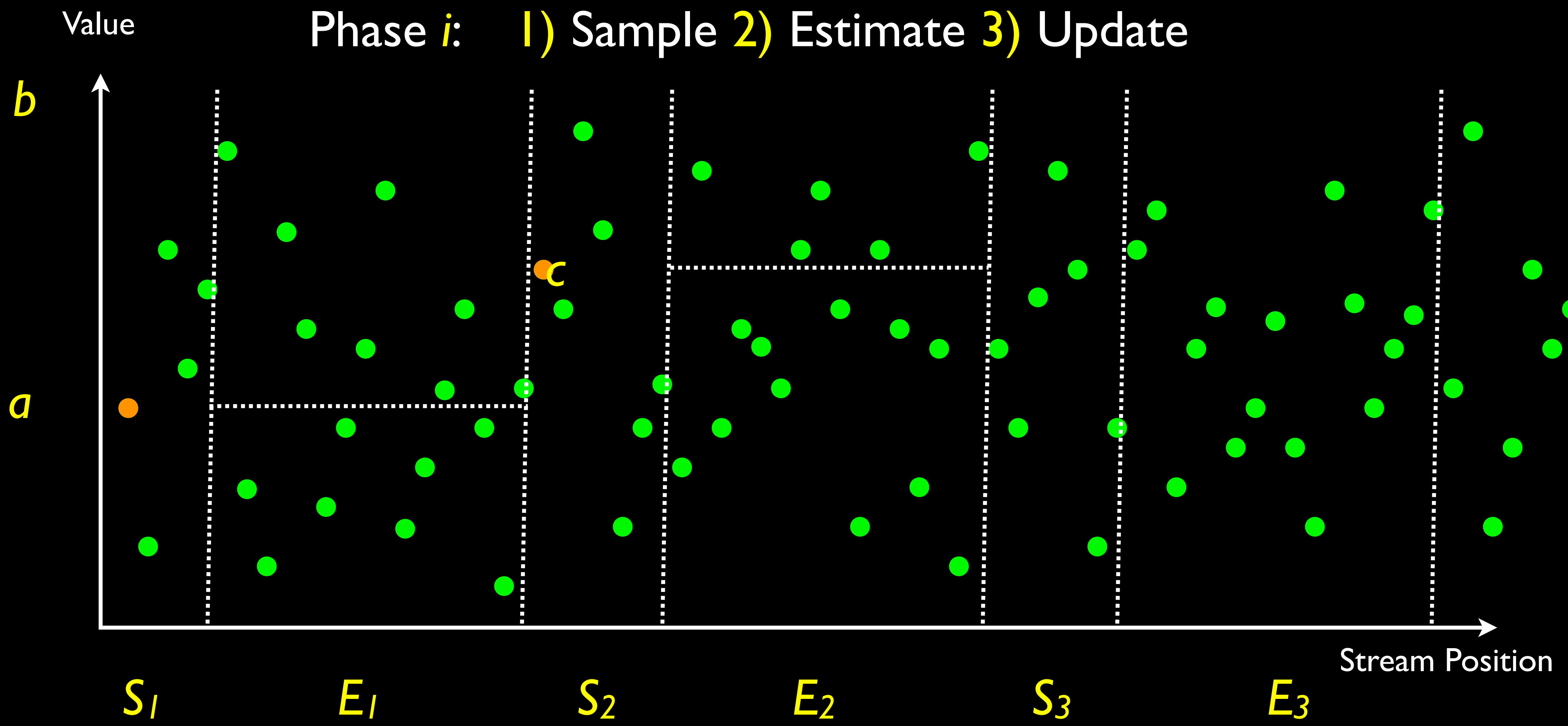

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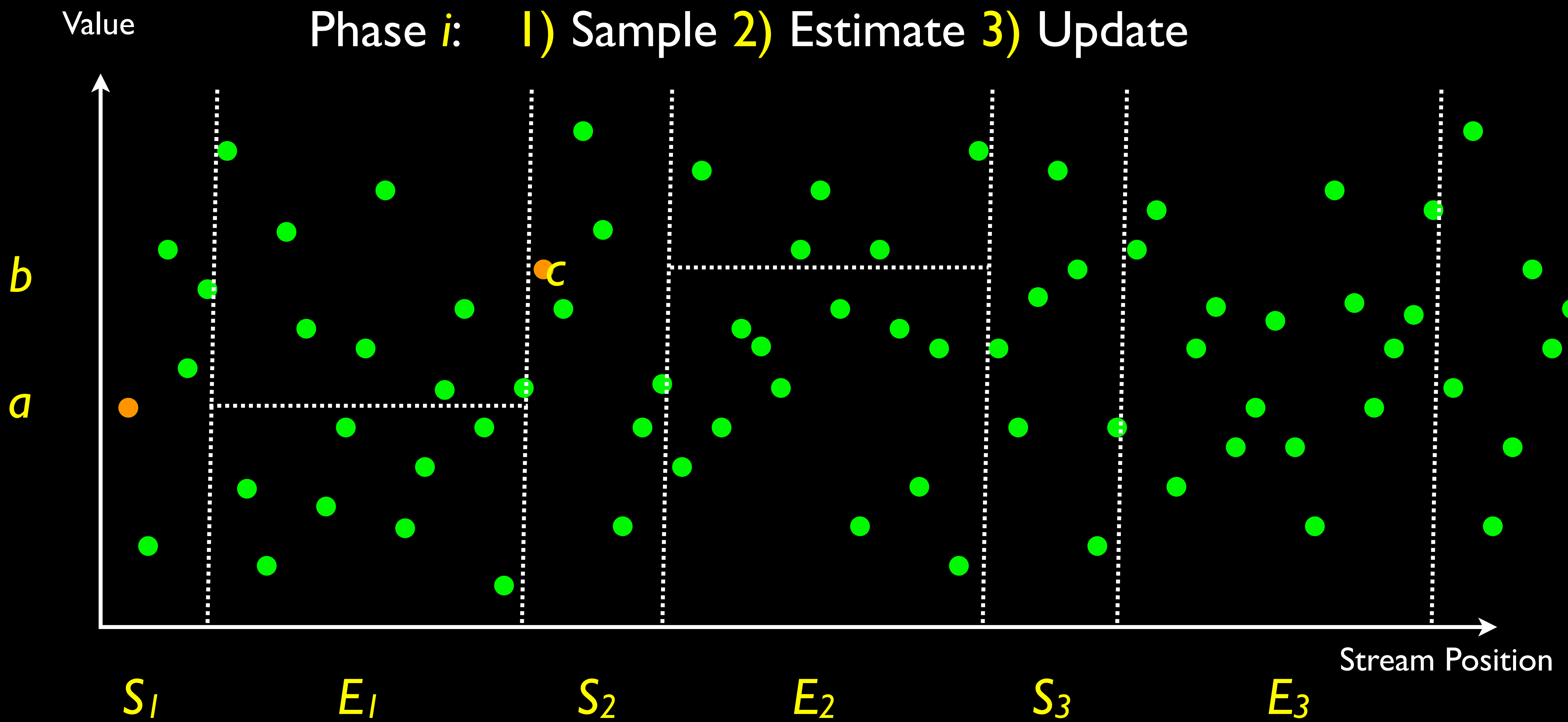

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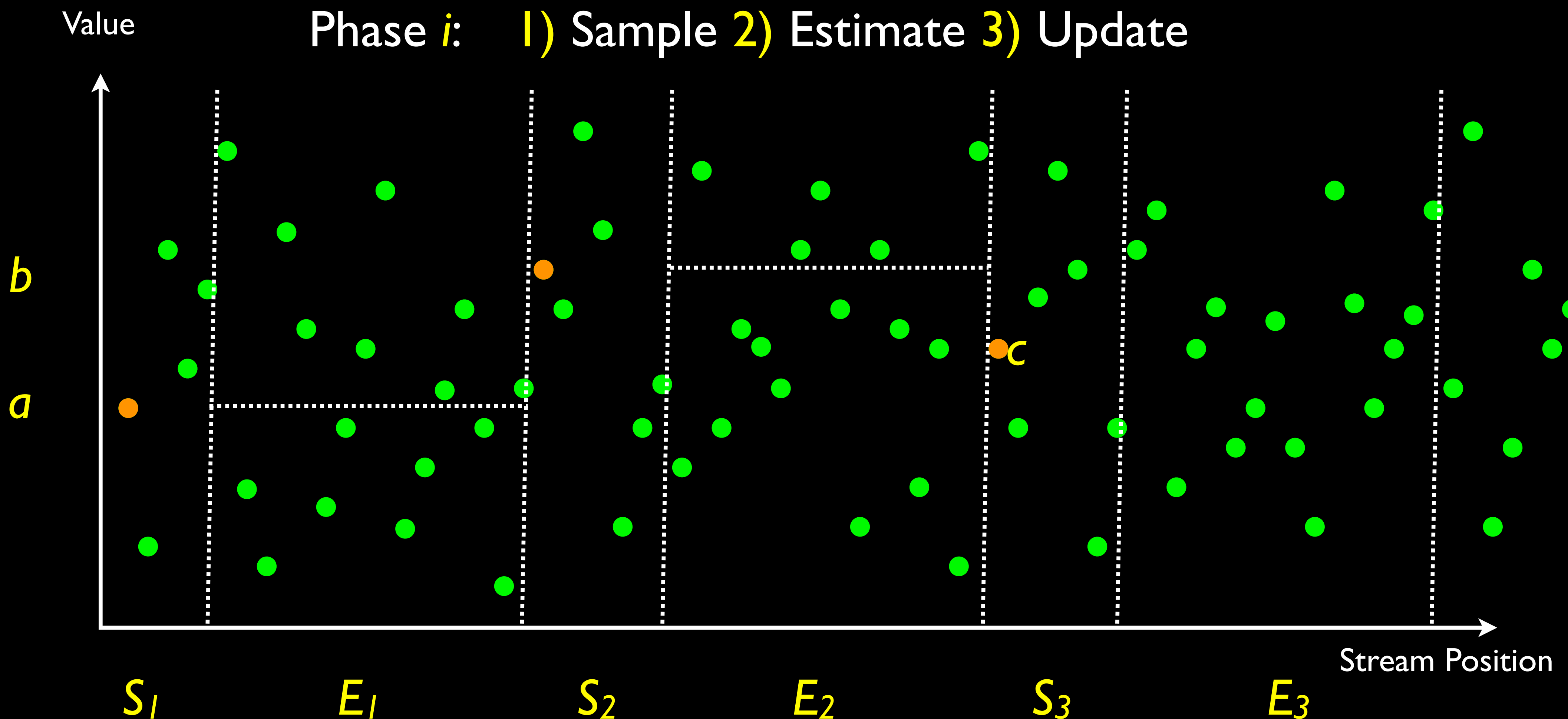

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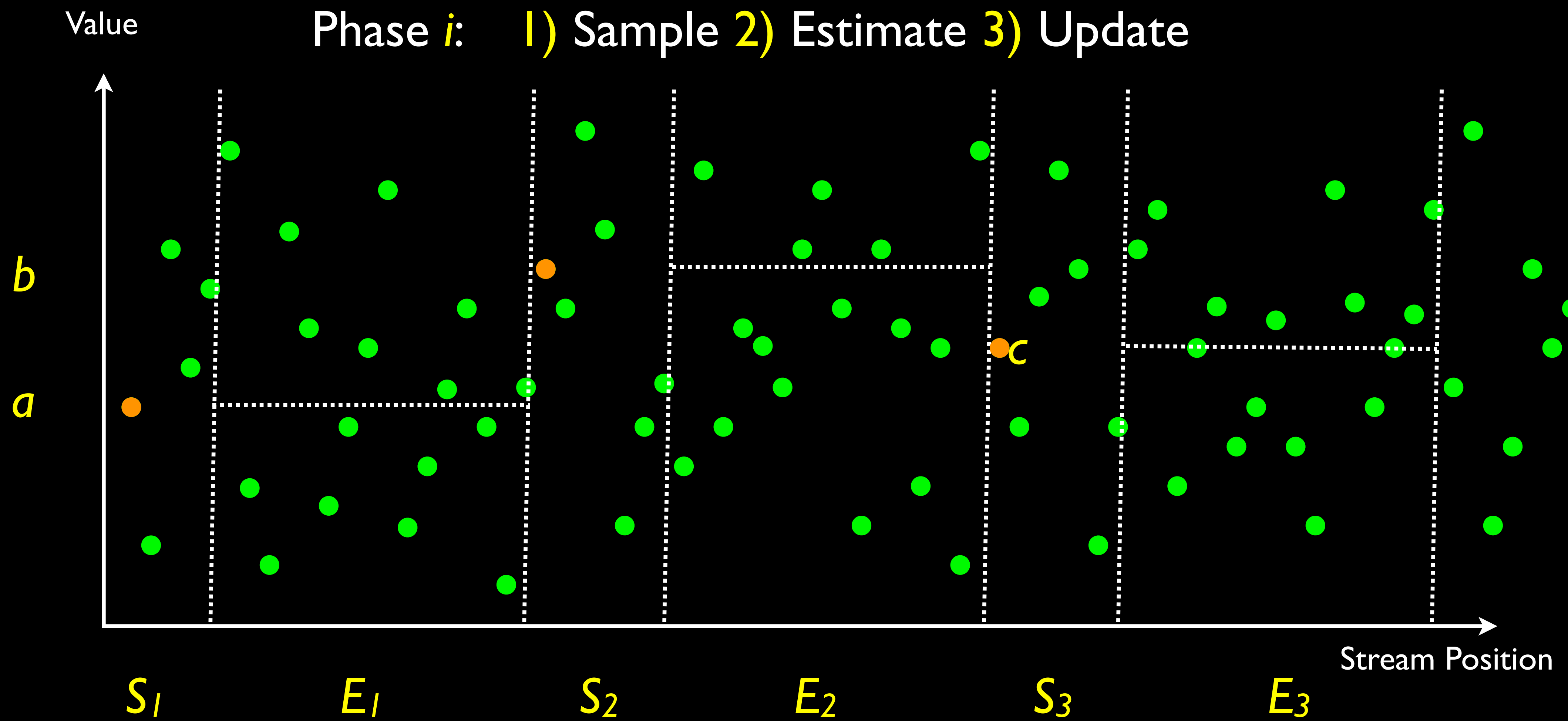

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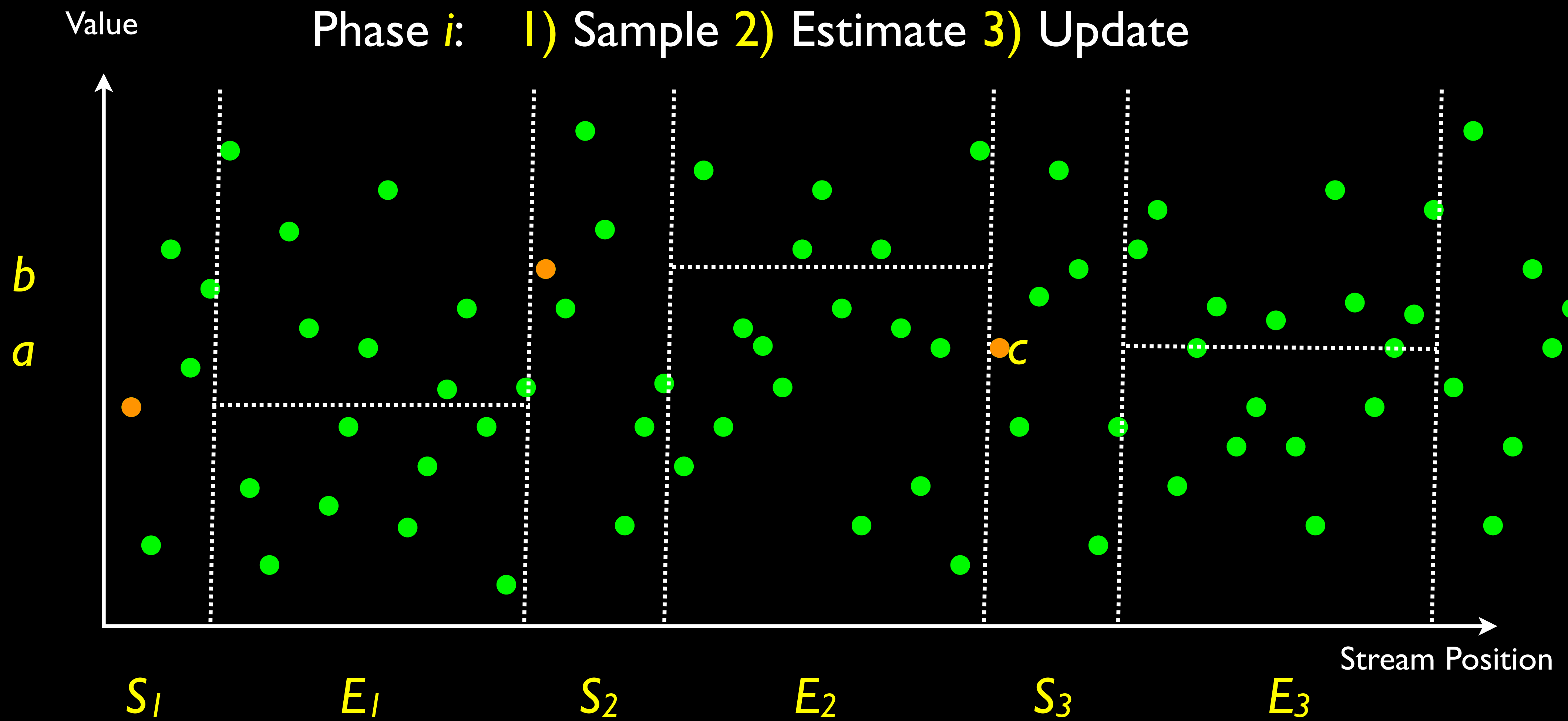

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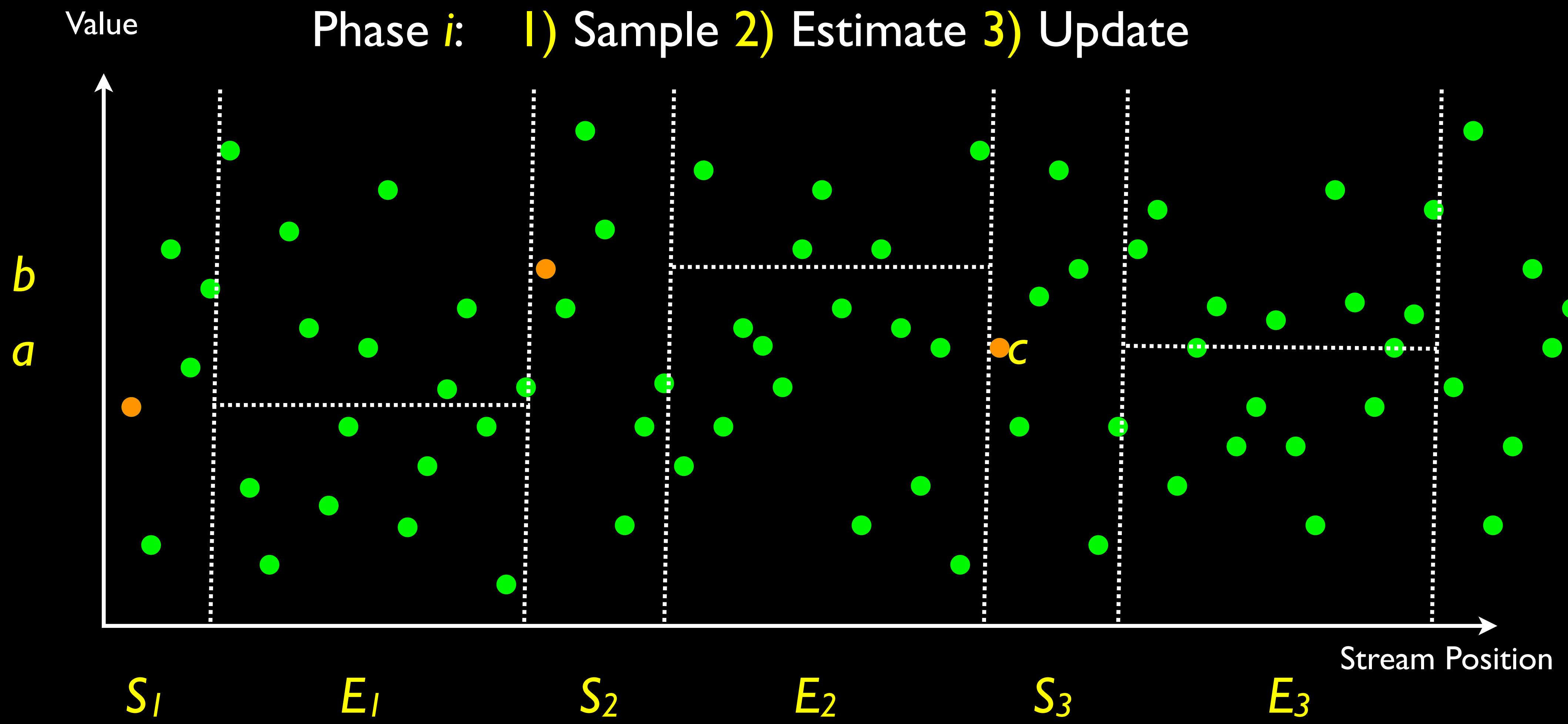

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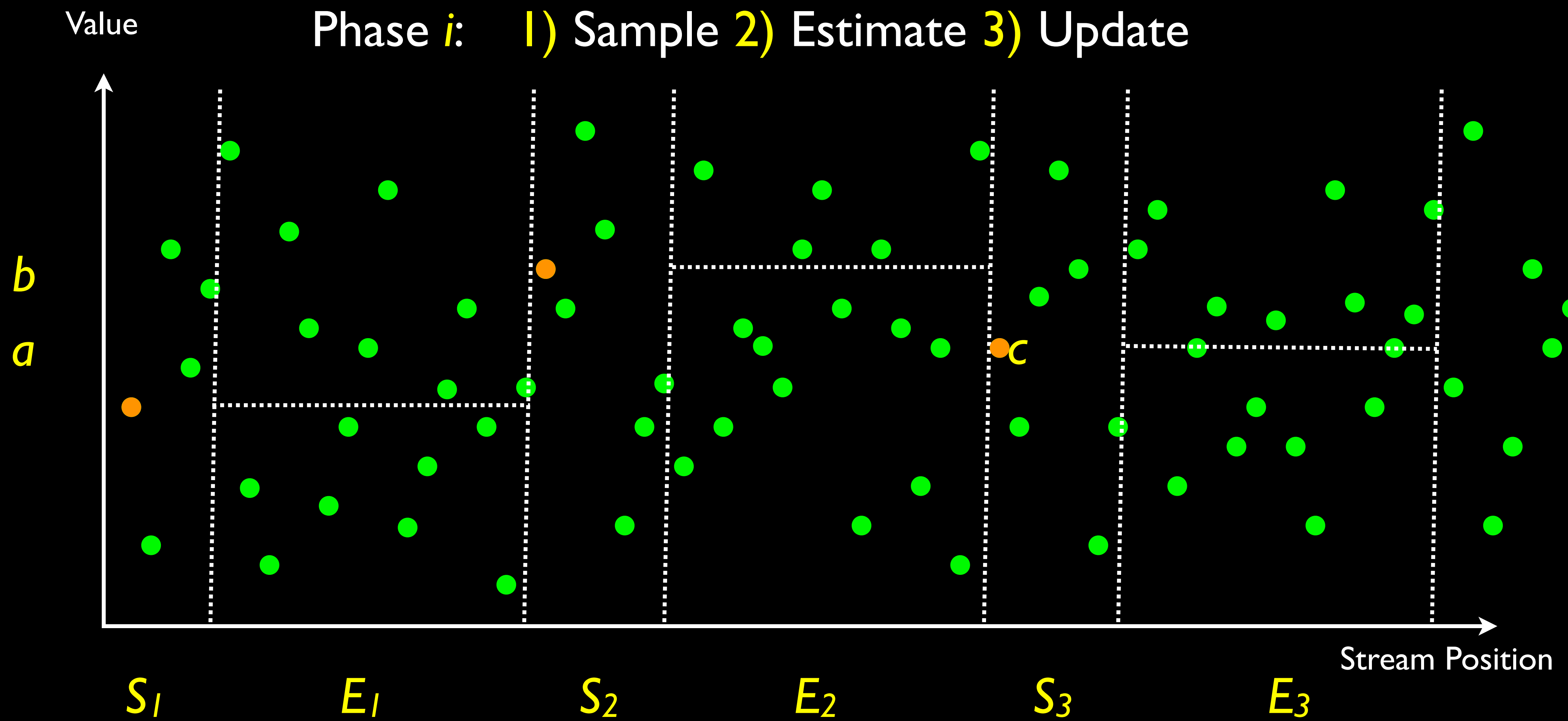

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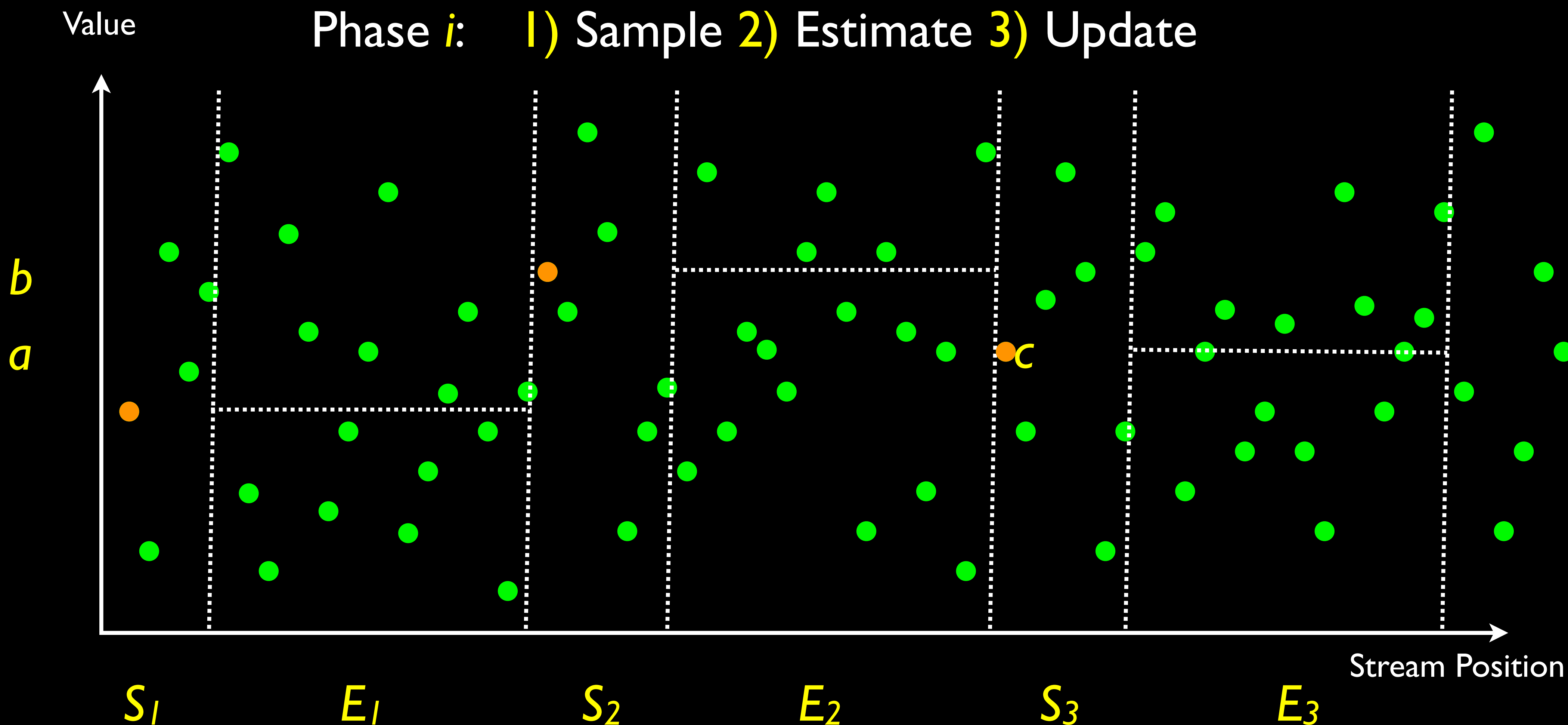

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If $|S_i|=O(\epsilon^{-1})$ and $\mu(a,b)=\Omega(\epsilon^{-1})$, then can find c in [a,b].

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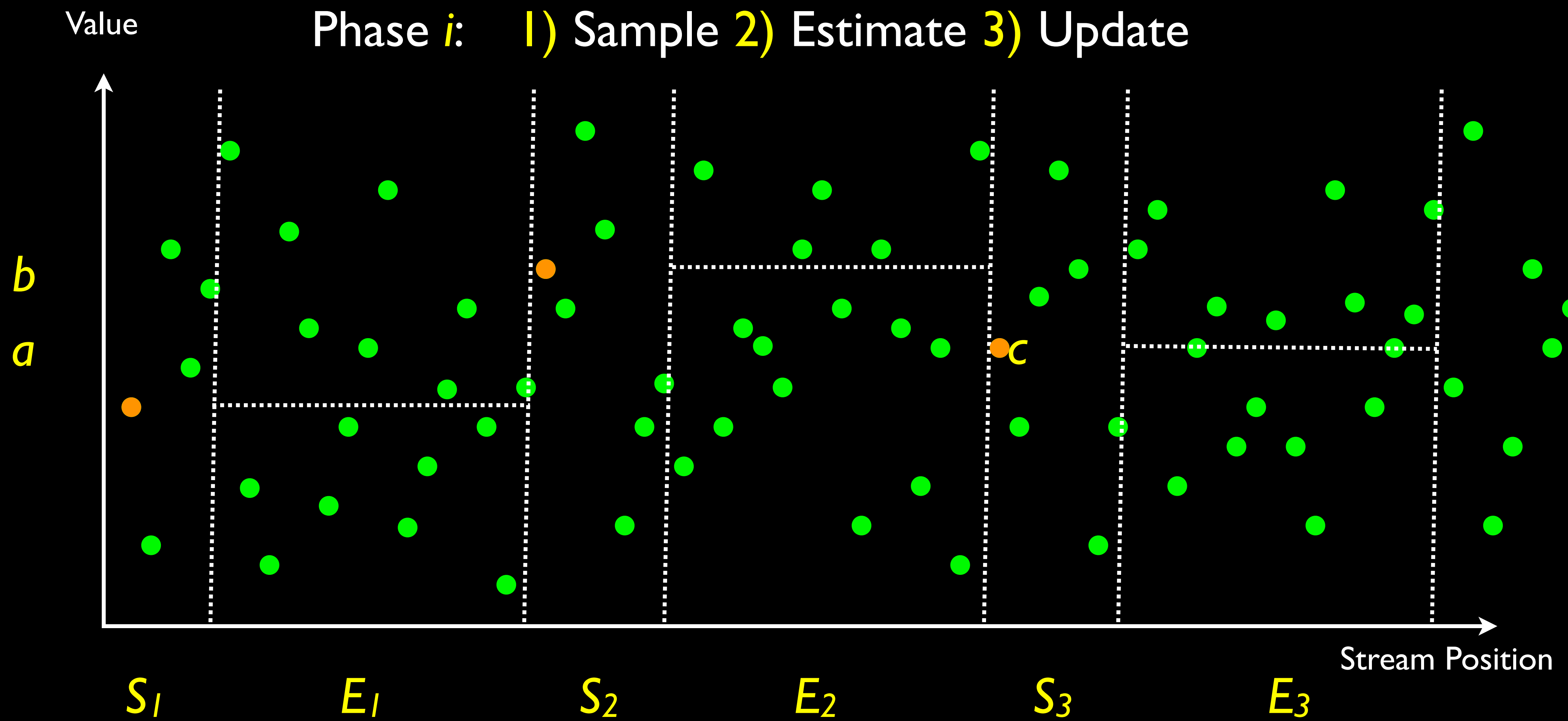

Analysis: If $|E_i|=O(\epsilon^{-2})$, we estimate $\mu(-\infty,c)$ up to $\pm\ \epsilon$.

If $\mu(-\infty,c) = 1/2 \pm \epsilon$, then return c.

If $|S_i|=O(\epsilon^{-1})$ and $\mu(a,b)=\Omega(\epsilon^{-1})$, then can find c in [a,b].

Expect $\mu(a,b)$ to half in each phase, hence $p = O(\log \epsilon^{-1})$

Algorithm: Maintain lower/upper bound [a, b] for median and c in [a,b]

Split stream in segments $S_1, E_1, S_2, E_2, \ldots, S_p, E_p$

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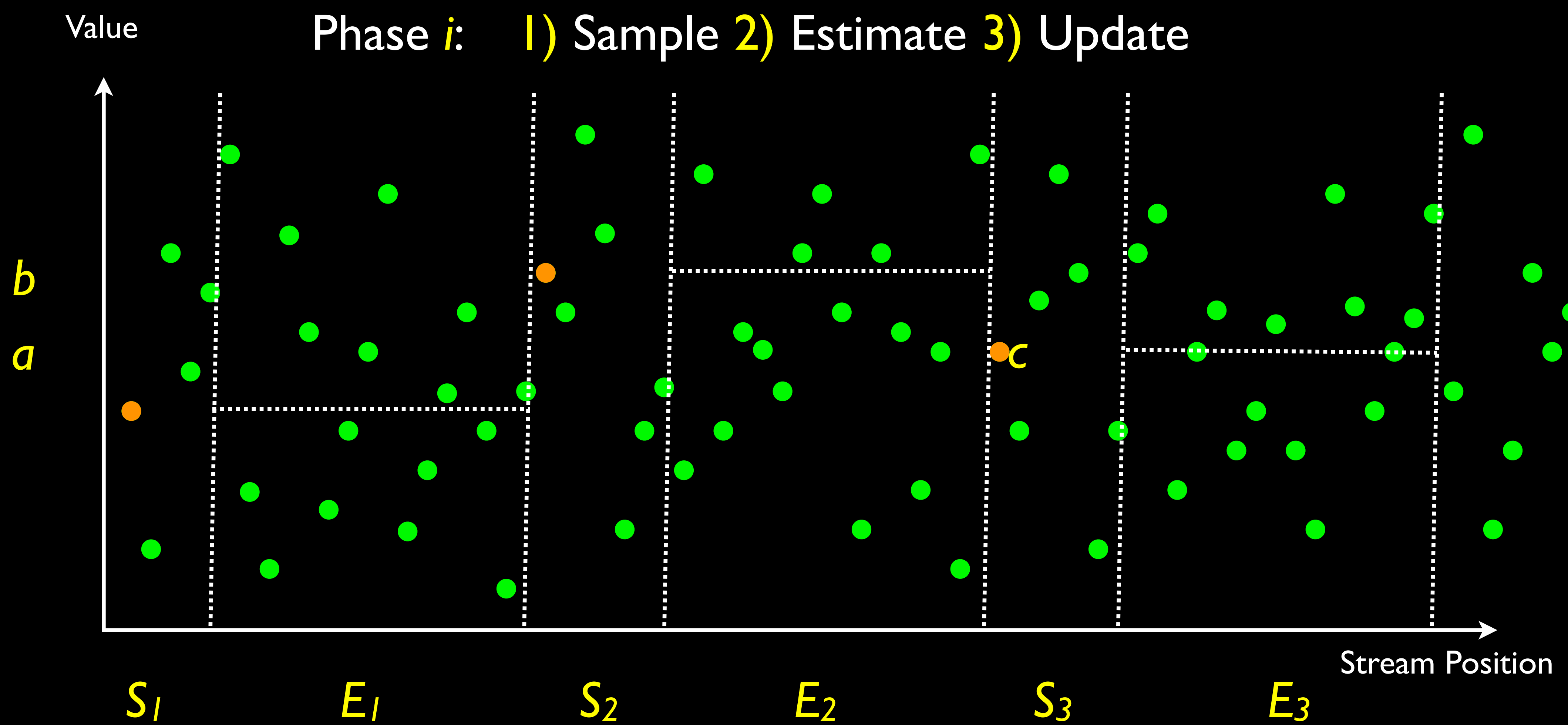

Thm: ϵ-approx median with $O(\epsilon^{-2}\log \epsilon^{-2})$ samples and $O(1)$ space

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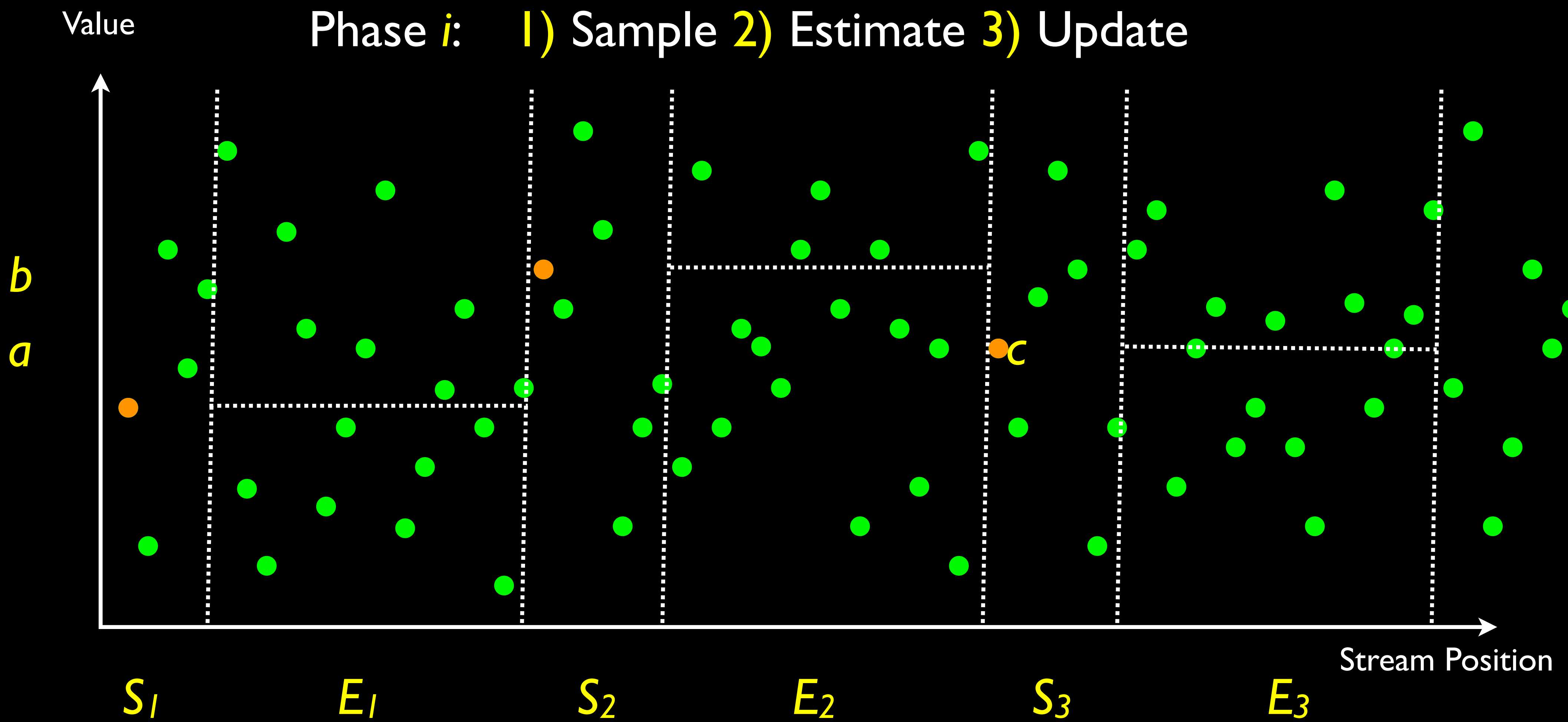

Thm: ϵ-approx median with $O(\epsilon^{-2}\log \epsilon^{-2})$ samples and $O(1)$ space

Thm: Given a length m stream *in random* order, can return an element with rank $m/2 \pm O(m^{1/2} \log^2 m)$ using $O(1)$ space.

Alice

length n
binary string x

Bob

index i in
range [n]

Alice

length n
binary string x

INDEX: "What's the value of x_i?"

Requires $\Omega(n)$ bits transmitted.

Bob

index i in
range [n]

Alice

length n
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INDEX: “What’s the value of x_i?”

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index i in
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- Assume there exists single-pass algorithm returning the median with prob. at least 3/4 using S space.

$2+x_1$ ... $2i+x_i$ ... $2n+x_n$

Alice

length n

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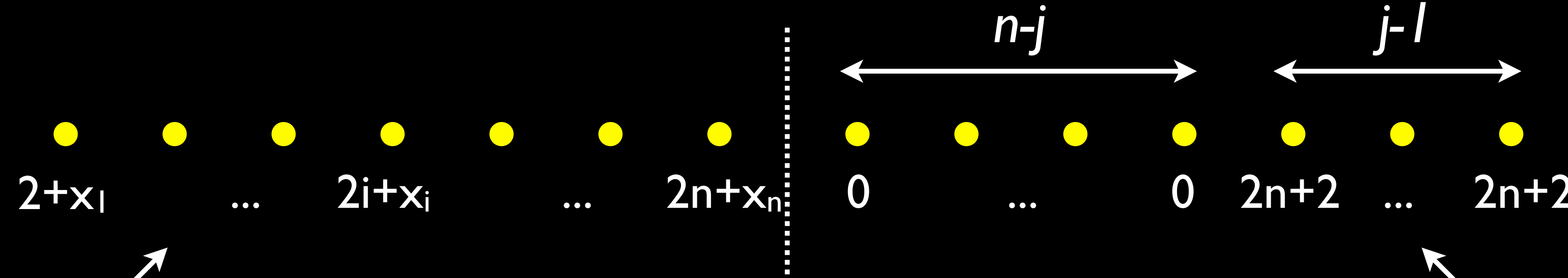

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n-j ⟷ *j-1*

2+x_1 … 2i+x_i … 2n+x_n | 0 … 0 2n+2 … 2n+2

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length n

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INDEX: "What's the value of x_i?"

Requires Ω(n) bits transmitted.

MEMORY STATE OF ALGORITHM

Bob

index i in

range [n]

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n-j ←→ j-1 ←→

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index i in
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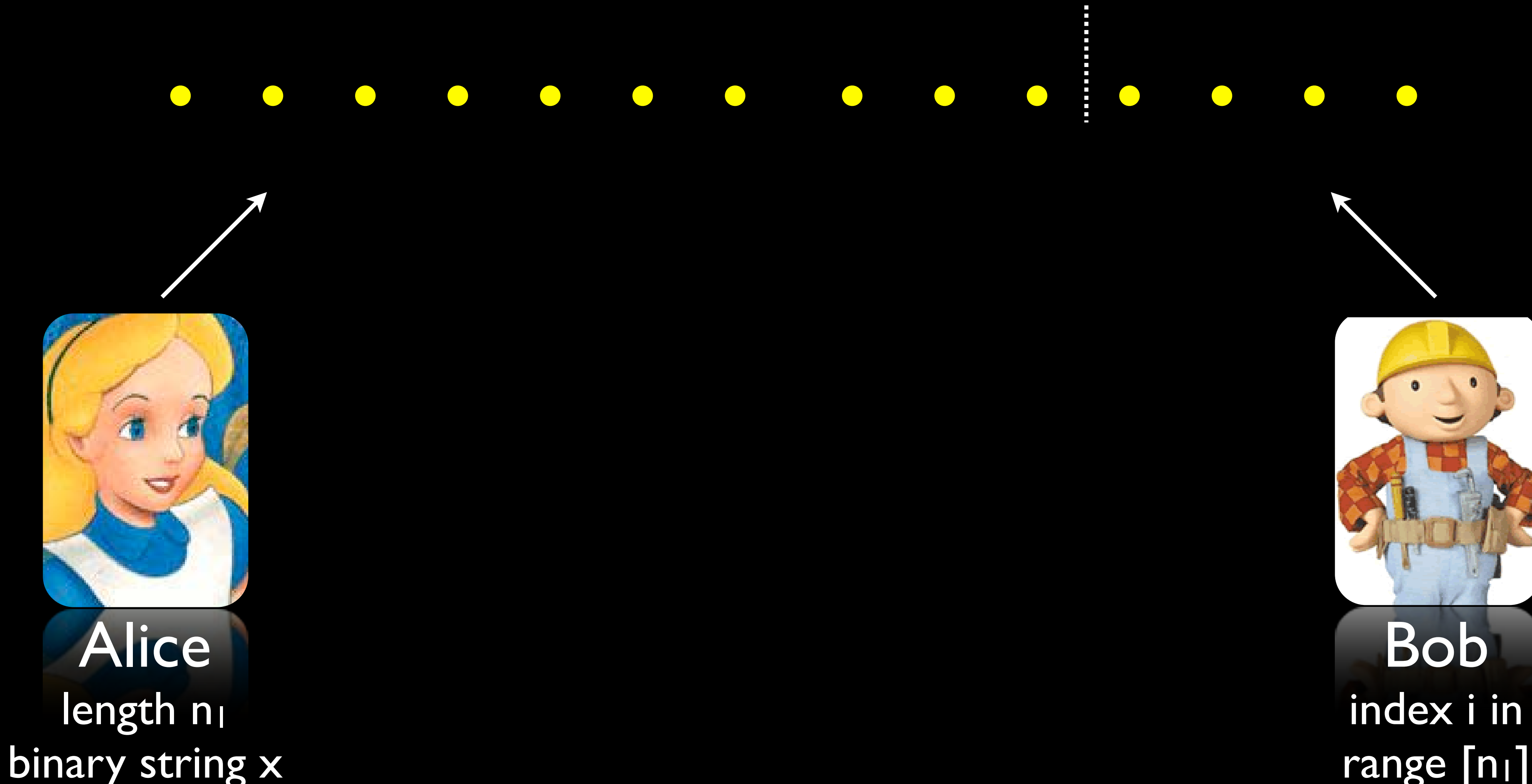
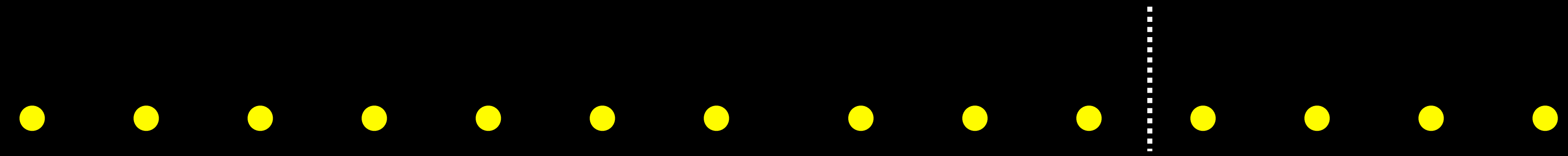
- Thm: S=Ω(n) [Henzinger, Raghavan, and Rajagopalan '99]


Alice
length n_1
binary string x
Bob
index i in
range [n_1]



Alice: picks b randomly from $[n_1]$ and inserts a random permutation of,

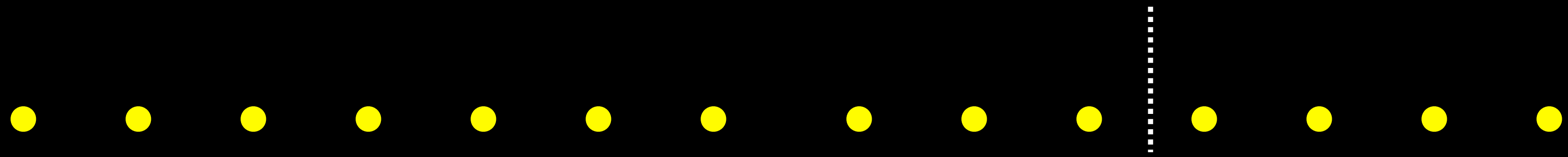
$$\{ \underbrace{0, \quad \ldots \quad, 0}_{(m-n_1-n_2-b)/2}, 2+x_1, \ldots, 2n_1+x_{n_1}, \underbrace{2n+2, \ldots, 2n+2}_{(m-n_1-n_2+b)/2} \}$$

Alice

length n_1 binary string x

Bob

index i in range $[n_1]$



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MEMORY STATE OF ALGORITHM and "b"

Alice

length n_1 binary string x

Bob

index i in range $[n_1]$

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Alice

length n_1

binary string x

MEMORY STATE OF ALGORITHM and "b"

Bob: inserts a random permutation of,

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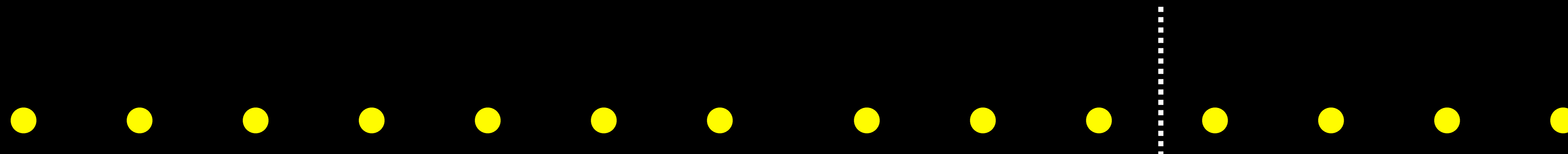
Bob

index i in range $[n_1]$



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Bob

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- If $n_1=o(\sqrt{n})$ and $n_1 n_2/n=o(1)$ then ordering is "close" to random.

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1. Models
2. Quantiles
3. Learning Distributions

Learning Distributions

Learning Distributions

- Stream: m samples a distribution with k piece-wise linear density function μ

Learning Distributions

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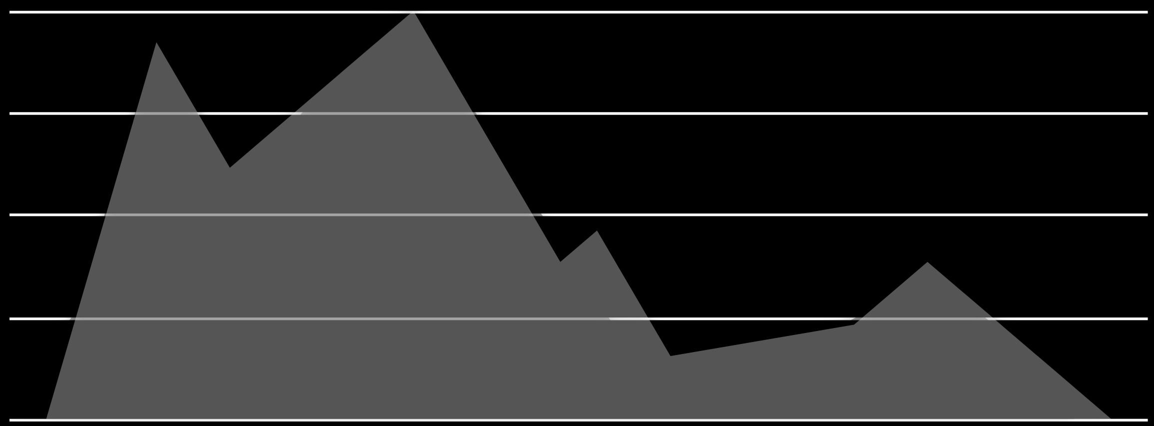
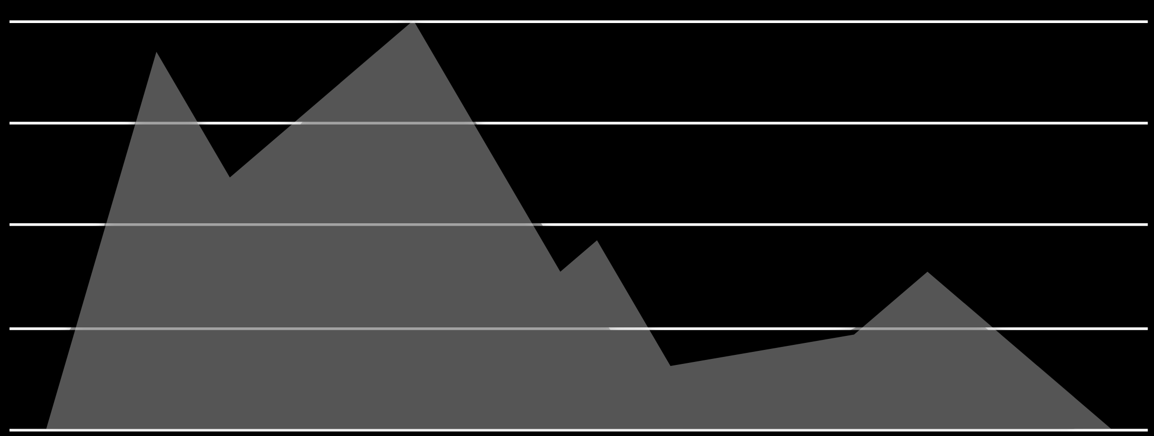


Learning Distributions

- Stream: m samples a distribution with k piece-wise linear density function μ
- Goal: Find k piece-wise linear density function μ' such that $|\mu - \mu'| < \epsilon$



Learning Distributions

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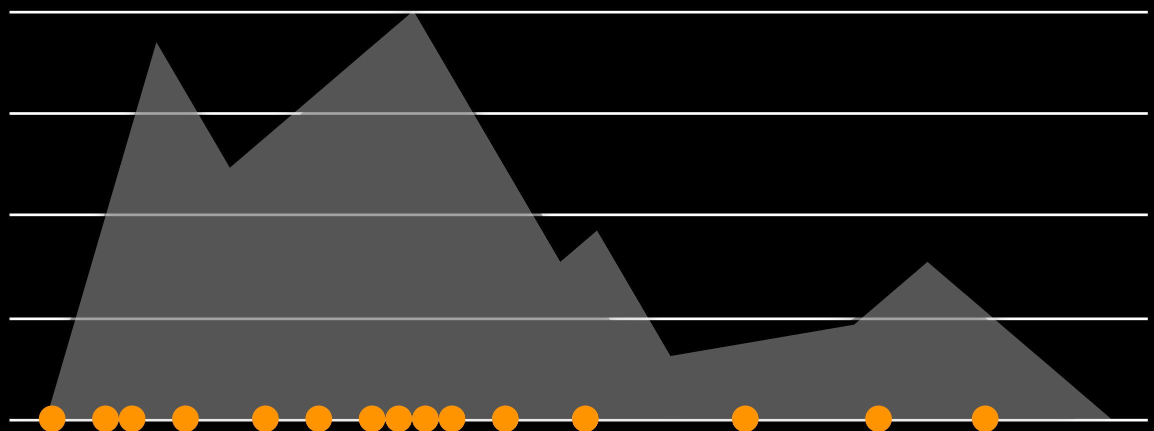

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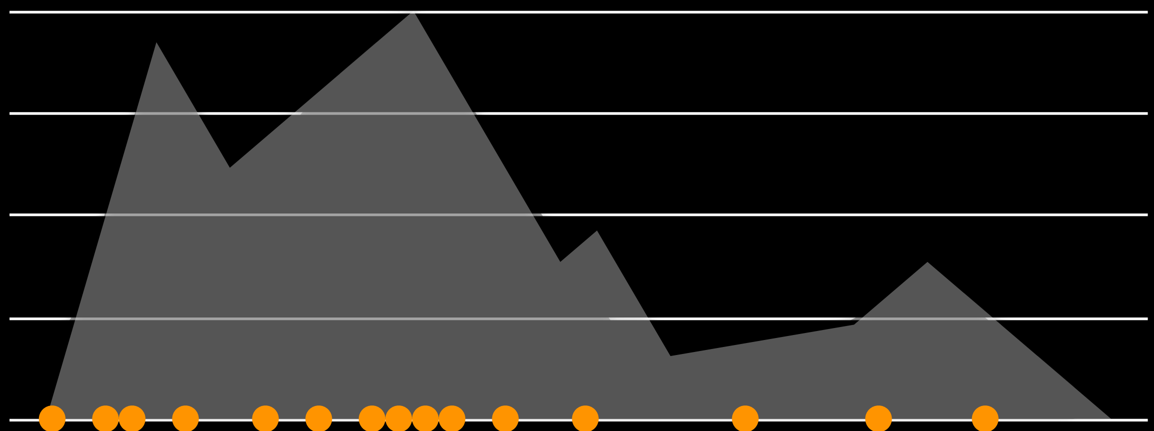

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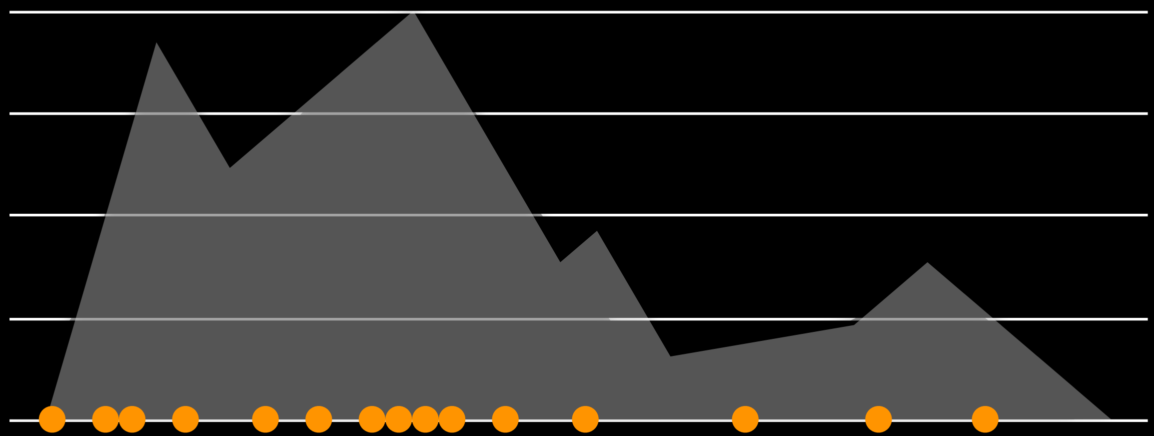
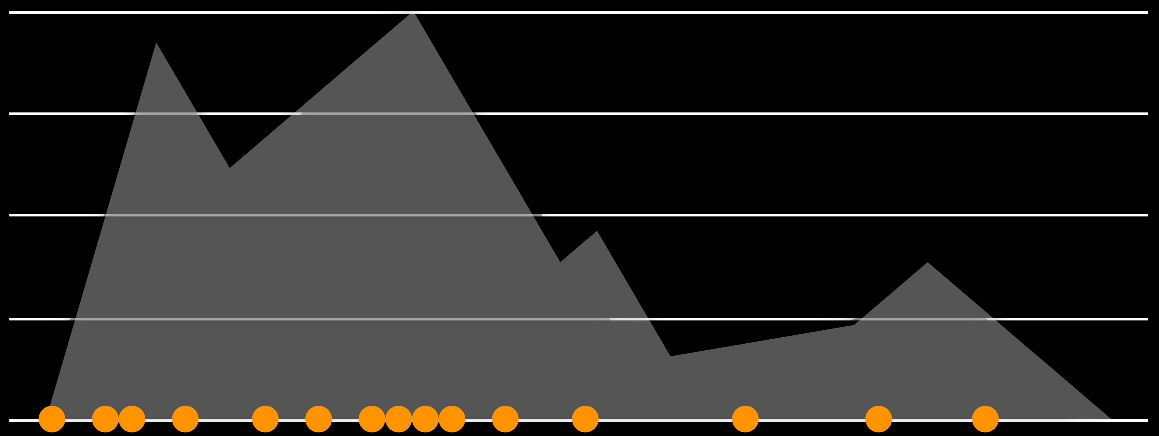
- Thm: $O(k^6 \epsilon^{-6})$ samples and $O(k^3 \epsilon^{-2/p})$ space with p passes. [Chang, Kannan '06]
- Thm: $O(k^2 \epsilon^{-4})$ samples and $O(k)$ space with one pass. [Guha, McGregor '06]

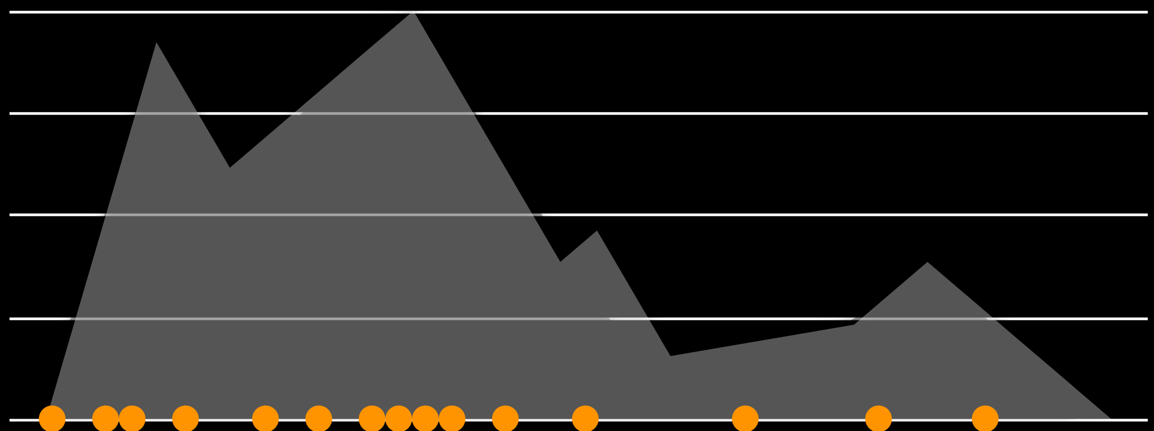


Learning Distributions



Learning Distributions

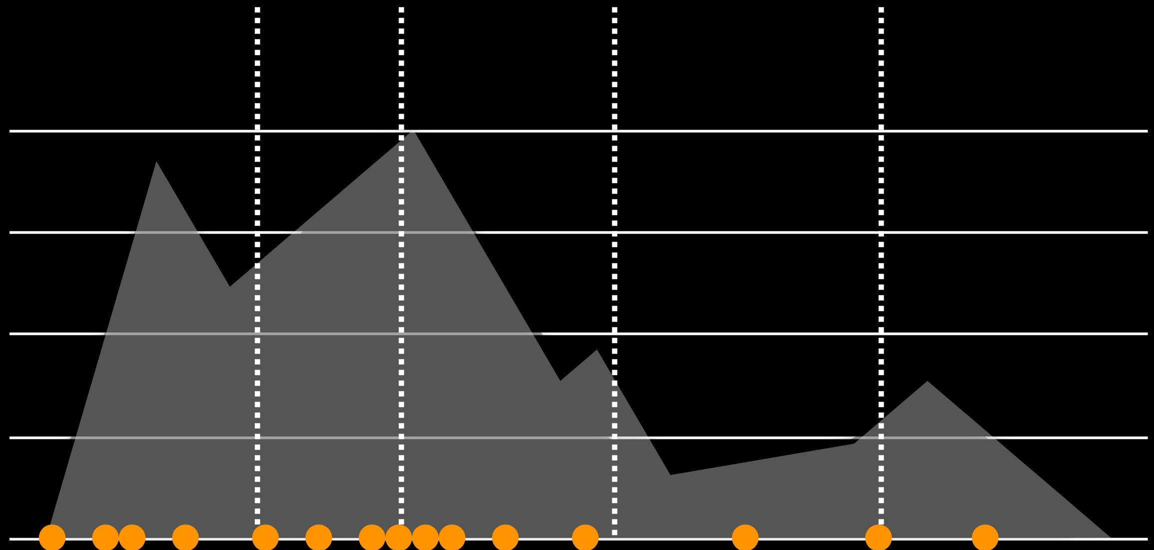
- Split into t_l intervals of approx equal mass



Learning Distributions

- Split into t_l intervals of approx equal mass

  $O(1/t_l^2)$ samples and quantile algorithm



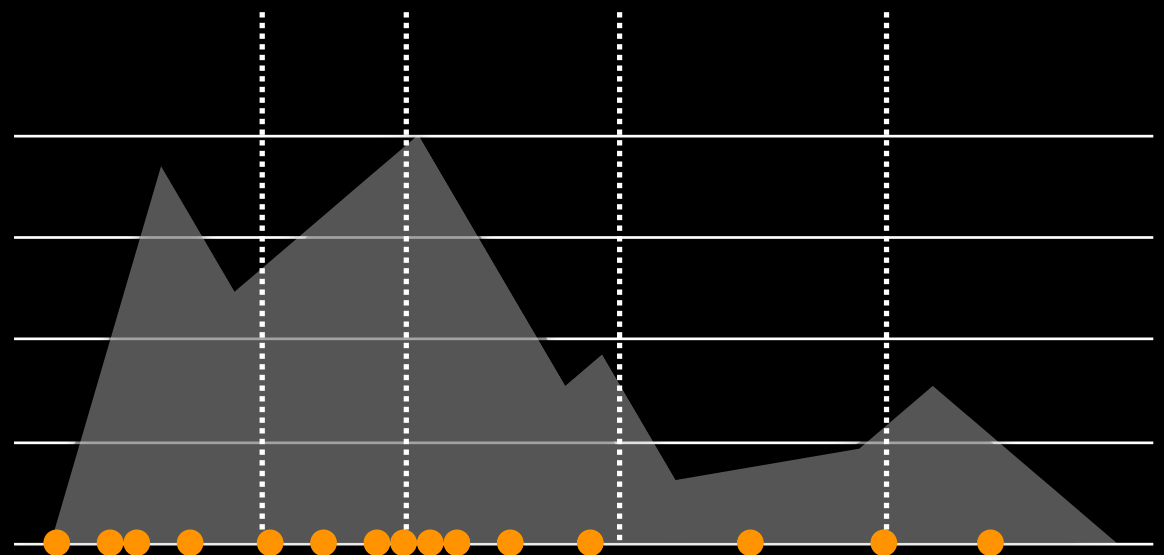
Learning Distributions

- Split into t_l intervals of approx equal mass

  $O(1/t_l^2)$ samples and quantile algorithm

- Test if μ conditioned on each interval [a,b] is β-far from linear

  $O(k/\beta^3)/\mu(a,b)$ samples, quantize, use L_1-sketch



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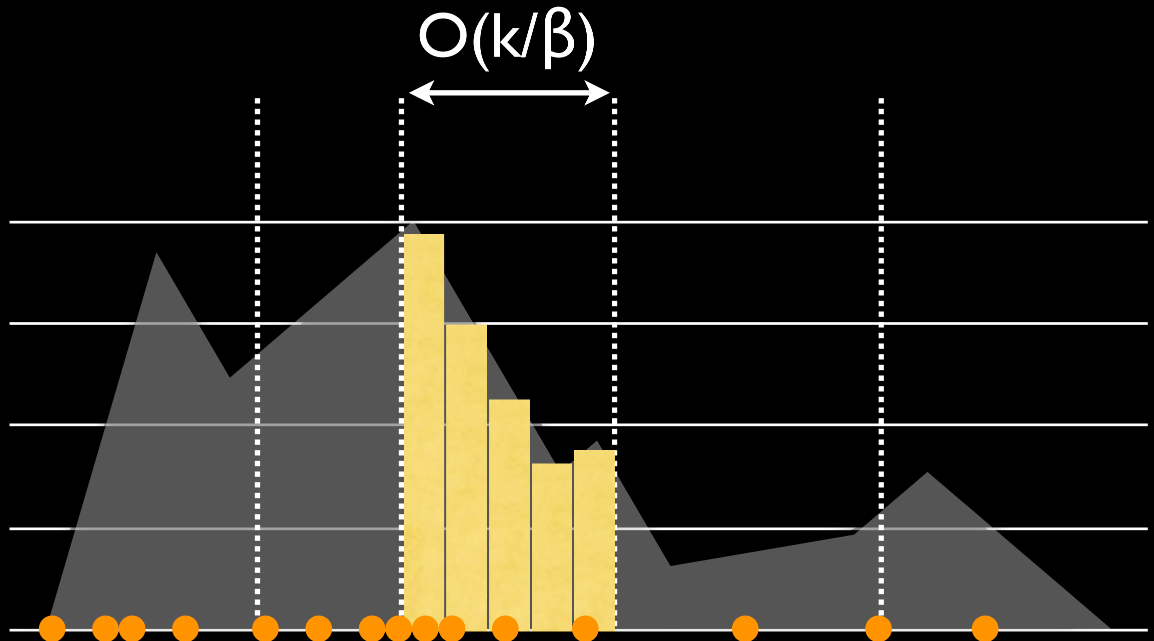

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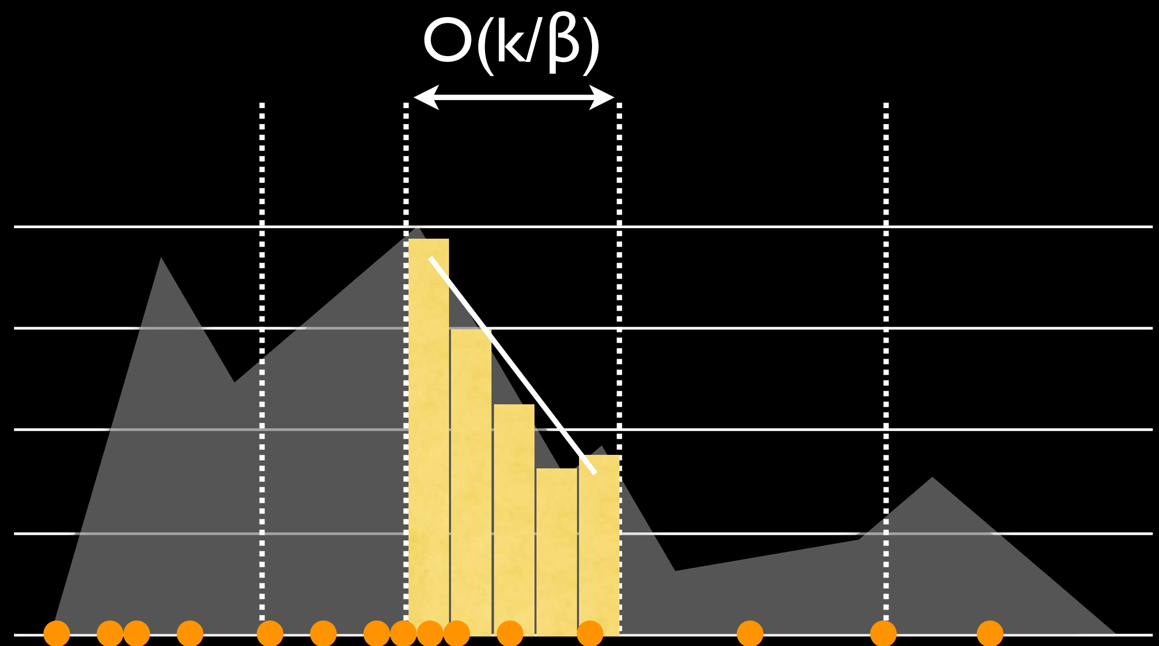

Learning Distributions

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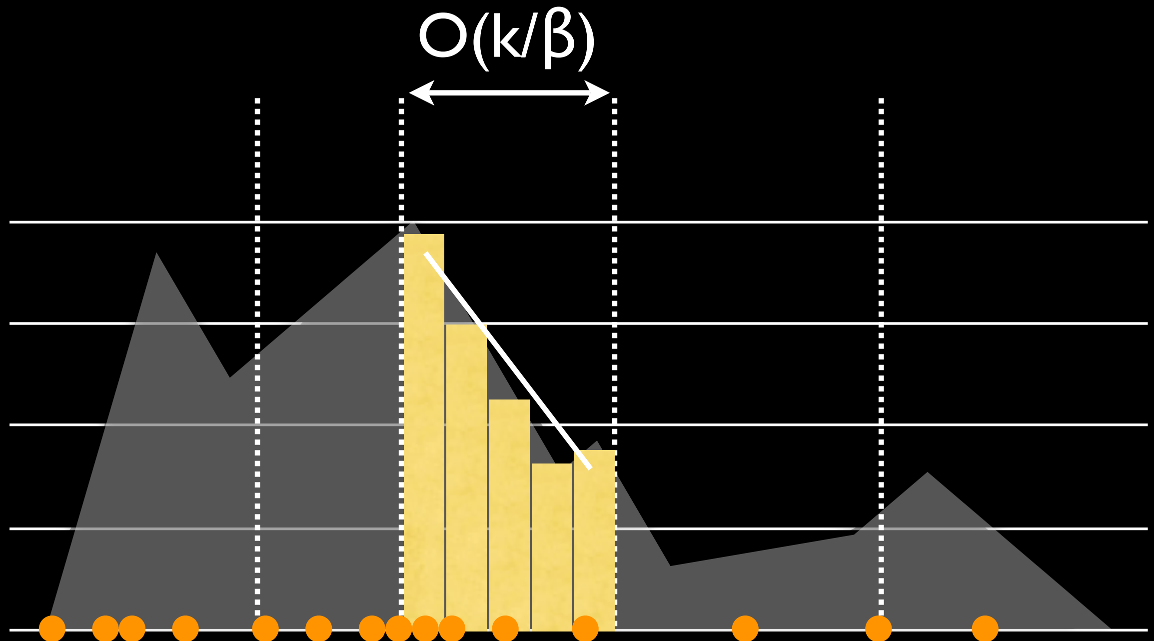
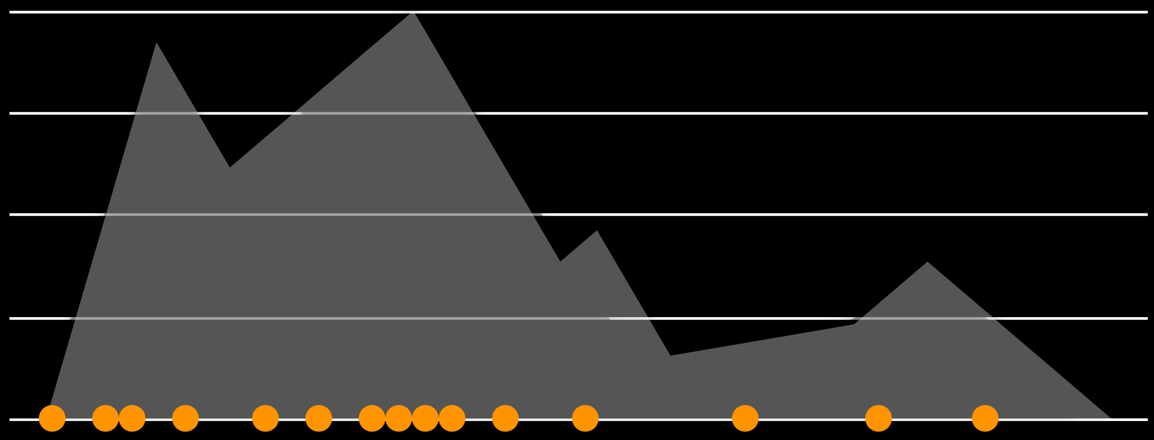
- Recurse on each non-linear interval



Learning Distributions



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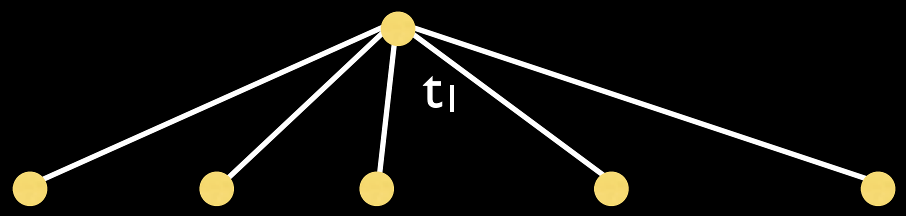

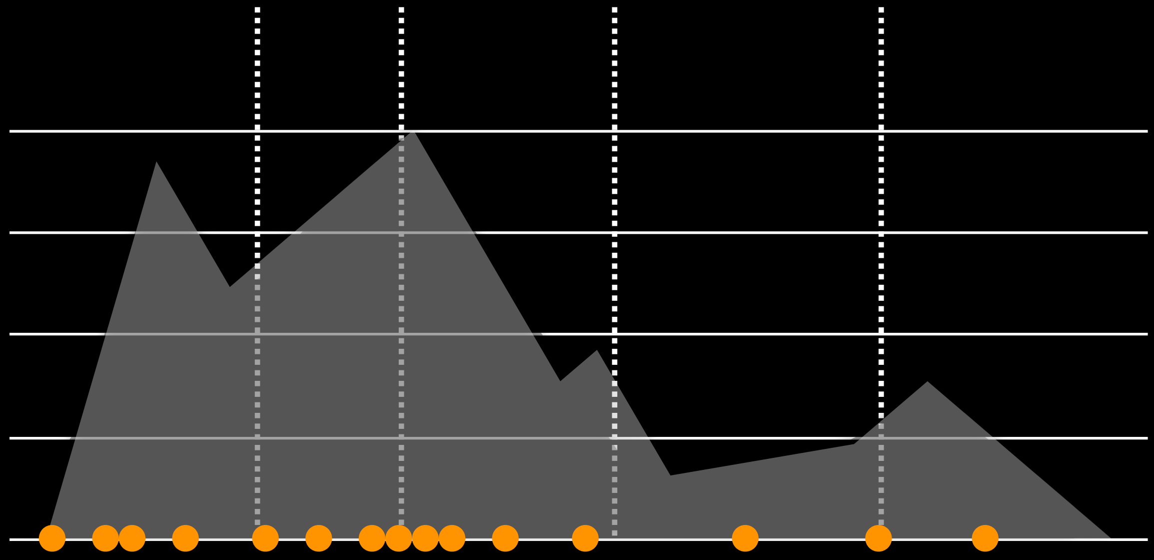

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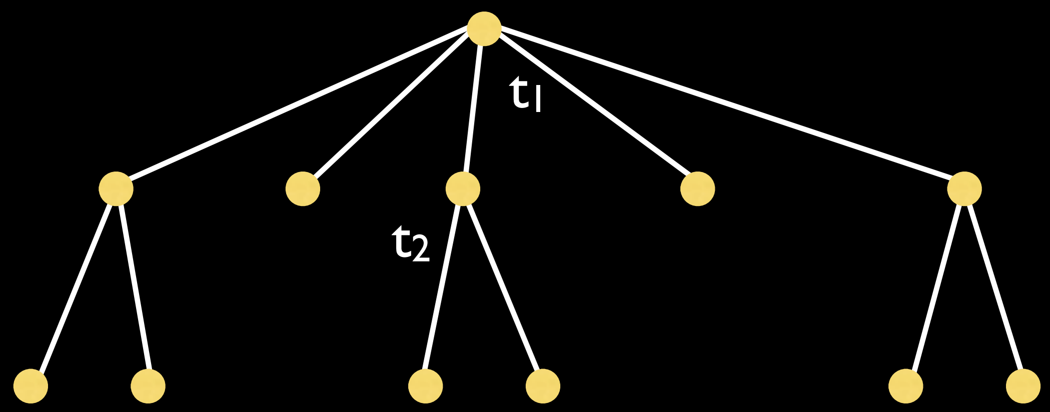
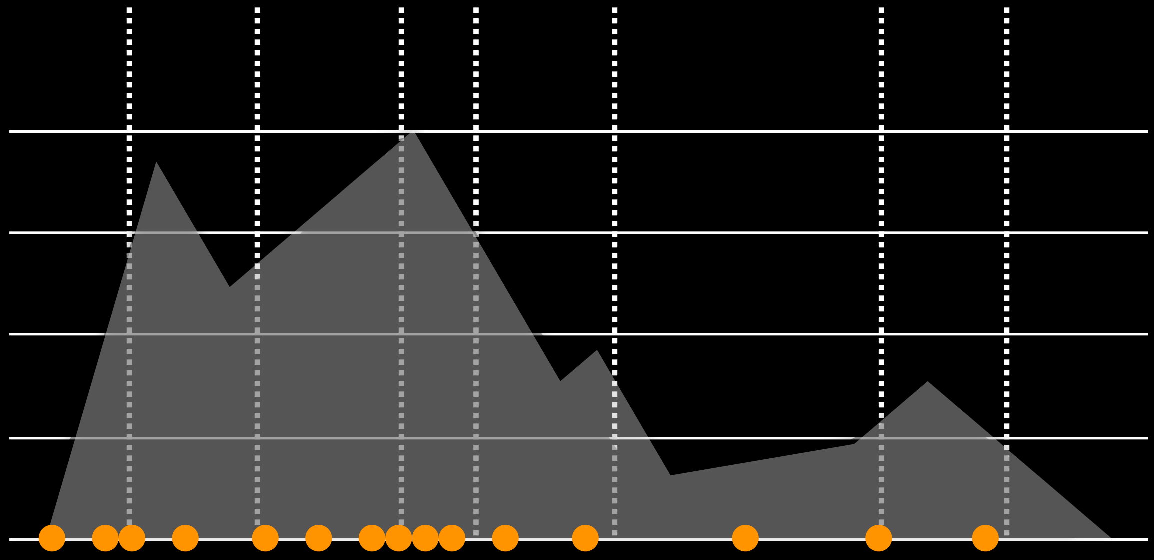
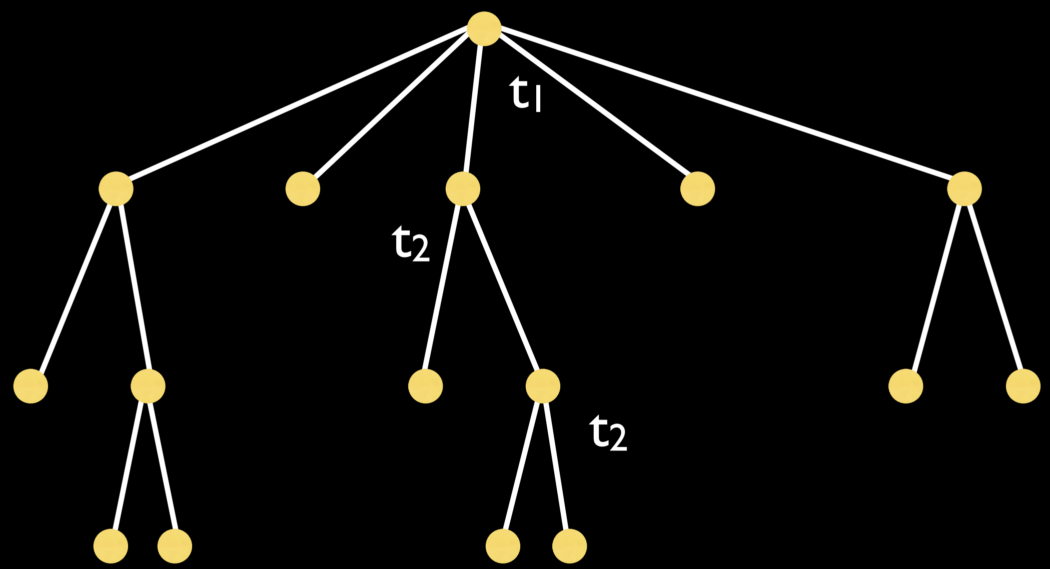
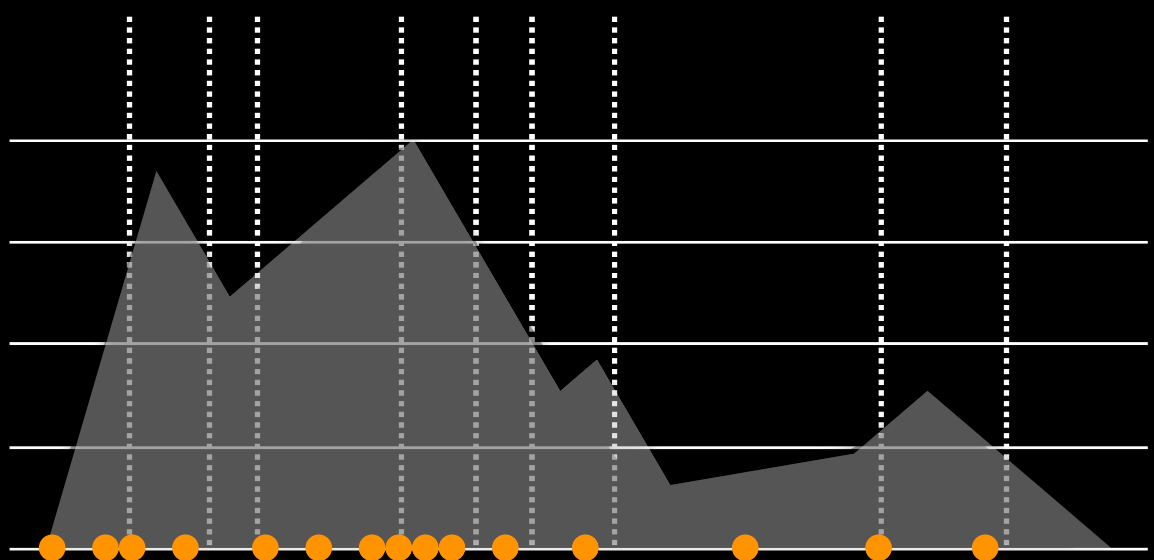




Learning Distributions





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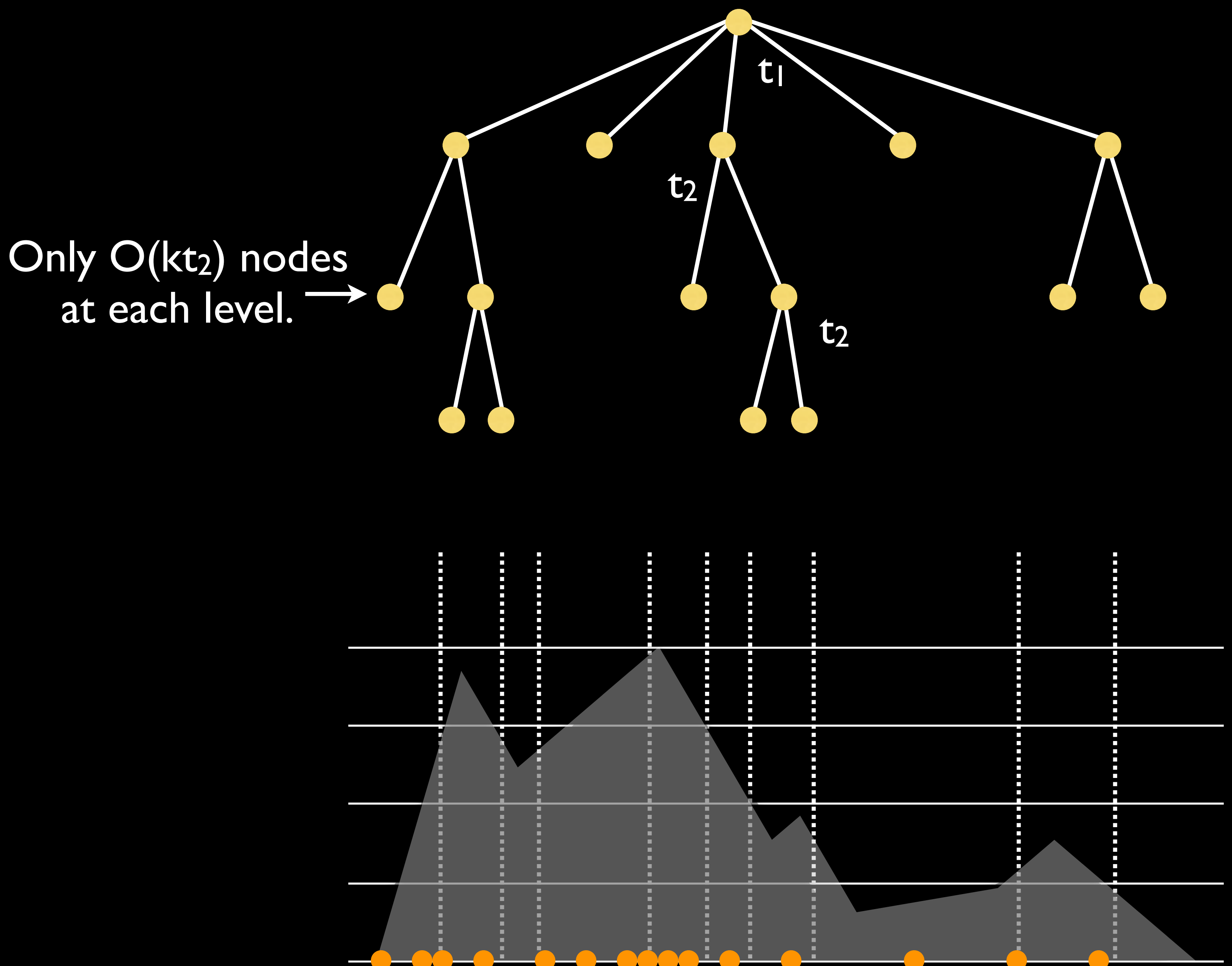

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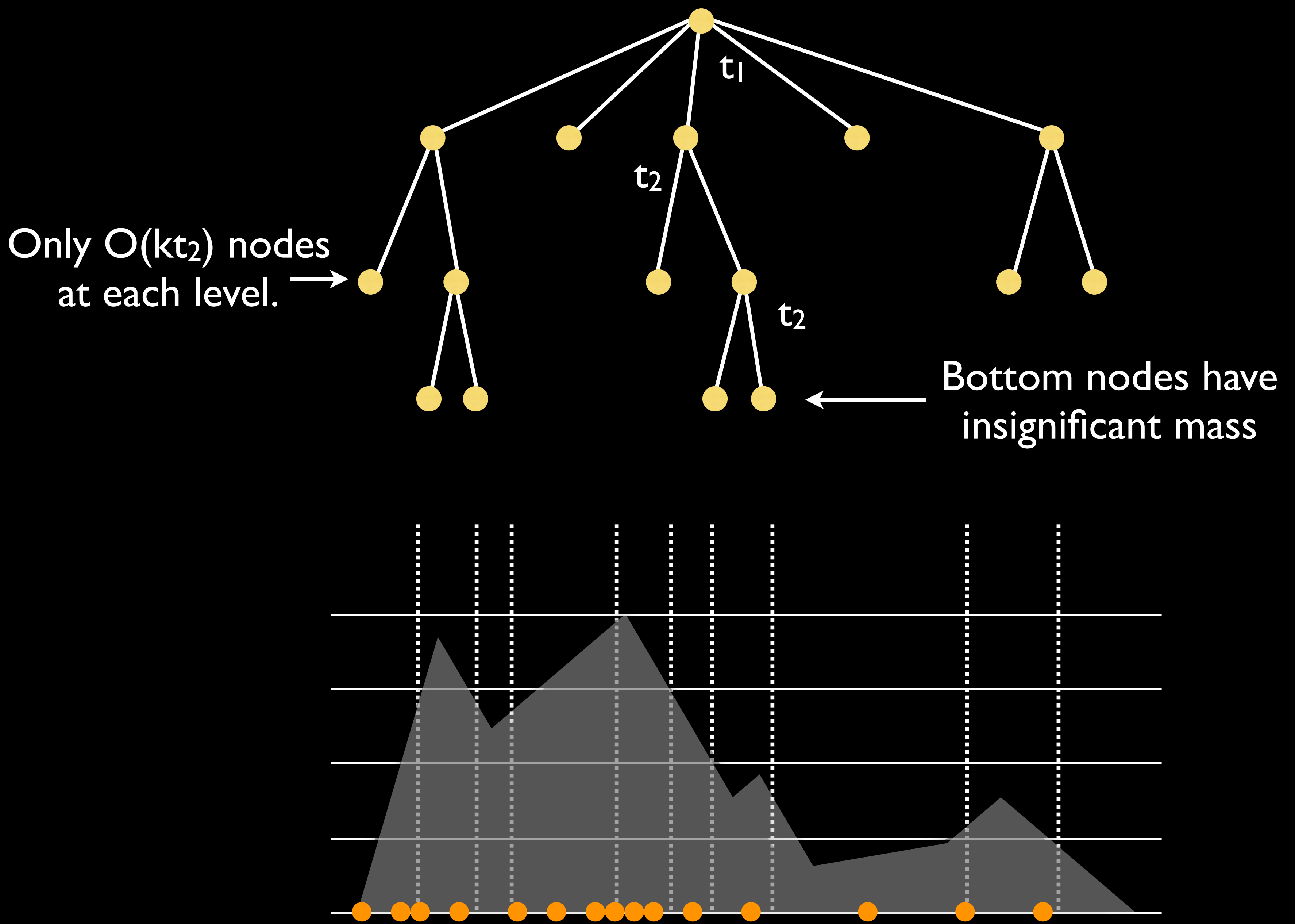

Summary

	Chang, Kannan '06	
Samples	$O(k^2 \epsilon^{-2})$	$O(k^6 \epsilon^{-6})$
Space	$O(k^2 \epsilon^{-2})$	$O(k^3 \epsilon^{-2/p})$
Passes	1	p
Re-order?	✔	✔

Summary

	Chang, Kannan '06		Guha, McGregor '06	
Samples	$O(k^2\ \epsilon^{-2})$	$O(k^6\ \epsilon^{-6})$	$O(k^2\ \epsilon^{-4})$	$O(k^2\ \epsilon^{-4})$
Space	$O(k^2\ \epsilon^{-2})$	$O(k^3\ \epsilon^{-2/p})$	$O(k)$	$O(k\ \epsilon^{-2/p})$
Passes	1	p	1	p
Re-order?	✔	✔	✖	✔

a) Estimating "*stochastic properties*" such as entropy and *f*-Divergences.

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b) Estimating expected values of aggregate properties given a "*probabilistic stream*."

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b) Estimating expected values of aggregate properties given a "*probabilistic stream*."

c) Learning about the source of a stream, i.e. "*upstream algorithms*."

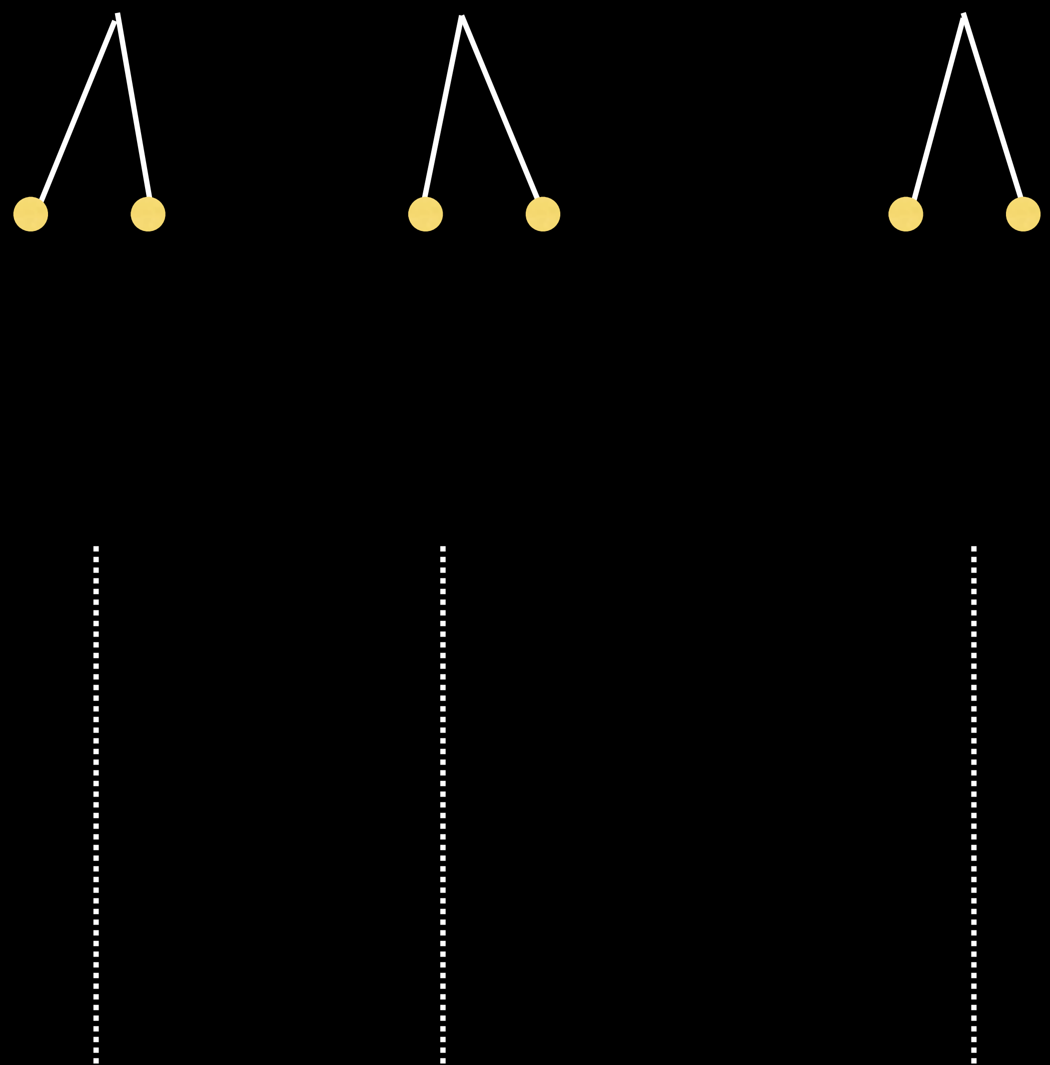
Examples:

sequence reconstruction, stream of iid samples, etc?

Algorithms:

quantiles/sufficient-statistics, piecewise-linear distributions, etc?

Questions?



1. Models
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3. Learning Distributions
4. Forgettron