

Homework 1

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Instructions Please compose your submission in L^AT_EX. Submit a softcopy by email to `satyadev@cse.iitk.ac.in`. Please follow the Academic Honesty Policy.

Due Date: January 28, 2014.

Questions

1. Write a decompressor algorithm for Move-to-Front coding. You do not have to write code, pseudocode will suffice.
2. Suppose you have the binary alphabet, and the bits are drawn uniformly at random. What is the expected length of the first run? What happens to the expected run length for larger alphabets?
3. Suppose a sequence of words is lexicographically sorted. Show that the longest common prefix of a word with its ancestors is obtained when comparing with its immediately preceding word.
4. Let X be an arbitrary random variable taking values in a finite set Ω . Show that $H(Y | X) = 0$ if and only if Y is a function of X .
5. (a) Show that a monotone decreasing sequence of real numbers always has a limit (the limit may be $-\infty$.)
(b) Show that any sequence of real numbers has a lim sup and a lim inf. *Hint: Use the result on monotone sequences.*
6. Using any of the notion lim sup, lim inf or lim, define the notion $f = O(g)$. [*Hint: If $f = O(g)$, does the limit $|f/g|$ always exist?*]
7. Show that $f(x) = \frac{1}{x}$ has a discontinuity at 0 and is continuous on all points in $\mathbb{R} - \{0\}$.
8. Show that every continuous bounded function $f : [0, 1] \rightarrow \mathbb{R}$ is Riemann-integrable.
9. Compute $\int_0^1 x^2 dx$ using the definition of Riemann integral. [*Hint: Write down a few lower Riemann sums $L^{(1)}, L^{(2)}, \dots, L^{(5)}$. Do you identify a pattern? Then take the limit as the number of cells in the partitions goes to infinity.*]
10. Show that if $A_1, A_2 \in \mathcal{F}$, then $\mu(A_1 \Delta A_2) = 0$ implies that $\mu(A_1) = \mu(A_2)$.