Course Logistics, Intro to Probabilistic Modeling and Inference

Piyush Rai

Topics in Probabilistic Modeling and Inference (CS698X)

Jan 7, 2019

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- **Course name:** Topics in Probabilistic Modeling and Inference (CS698X) - “TPMI” or just “PMI”
- **Timing and Venue:** M/W 17:10-18:25, KD-101

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- Auditing? Don't need formal permission from me. Send me email to be added to the mailing list

The TA Team

- TA office hours/locations and contact details will be posted on Piazza


Shivam Bansal


Dhanajit Brahma


Sunabha Chatterjee


Abhishek Kumar


Siddhartha Saxena


Vinay Kumar Verma

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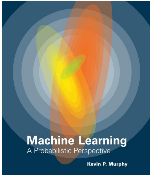
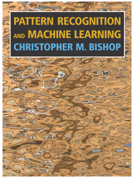
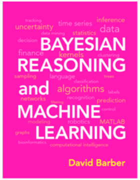
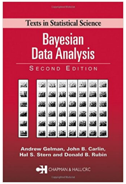
  - Kevin Murphy, Machine Learning: A Probabilistic Perspective (MLAPP), The MIT Press, 2012.
  - Christopher Bishop, Pattern Recognition and Machine Learning (PRML), Springer, 2007.
  - David Barber. Bayesian Reasoning and Machine Learning (BRML), Cambridge Univ. Press, 2012.
  - Andrew Gelman *et al.* Bayesian Data Analysis (BDA), Chapman & Hall/CRC, 2013









Background Expected (Important)

- Basic concepts from probability theory (also refer to the prob-stats refresher on course webpage)
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  - Gradient descent, stochastic gradient descent, alternating optimization
  - Basic optimization algos for latent variable models (e.g., expectation maximization)

Probabilistic Modeling and Inference

(or living happily with uncertainty)

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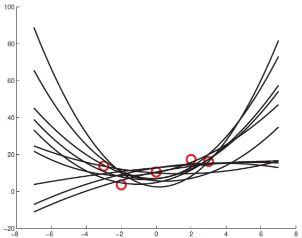
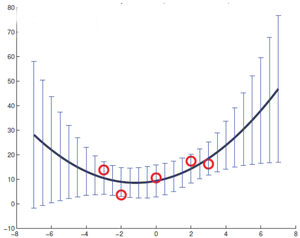
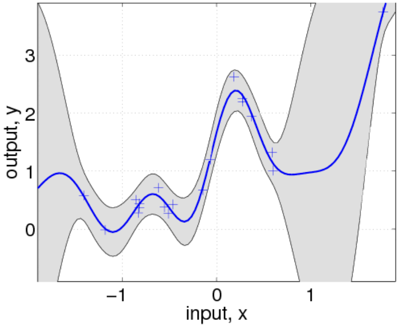
- We may have imprecise/noisy data. Need to model the noise/uncertainty explicitly
  - Can do it using appropriate probability distributions
- Due to data scarcity, there may be uncertainty in the estimated model parameters and predictions
  - Can do so by learning a probability distribution over parameters and predictions





Why a Probabilistic Approach (Contd)?

- Sequential decision-making: Estimate of model's uncertainty can "guide" us, e.g.
  - Given the current estimate of a function uncertainty over the input space, where should we acquire the next observation?



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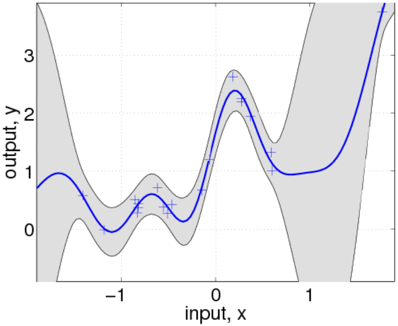
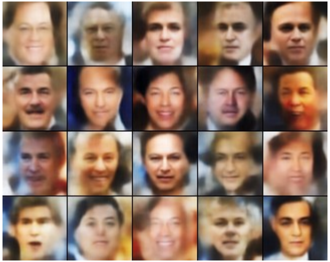


- This has many applications in active learning, reinforcement learning, Bayesian optimization, etc.

Why a Probabilistic Approach (Contd)?

- Sometimes we may be interested in learning the underlying probability distribution of data
- Learning the distribution can enable us to understand and also generate new data!



Modeling Data Probabilistically: A Simplistic View

- Assume data $\mathbf{X} = \{\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N\}$ generated from a probabilistic model with unknown parameters θ

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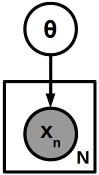

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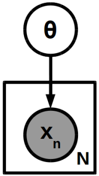

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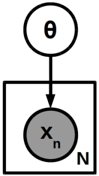

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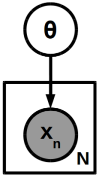

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- Can use the learned model to make predictions
  - E.g., the probability $p(\boldsymbol{x}_*|\theta)$ or $p(\boldsymbol{x}_*|\mathbf{X})$ of a new input $\boldsymbol{x}_*$ under this model

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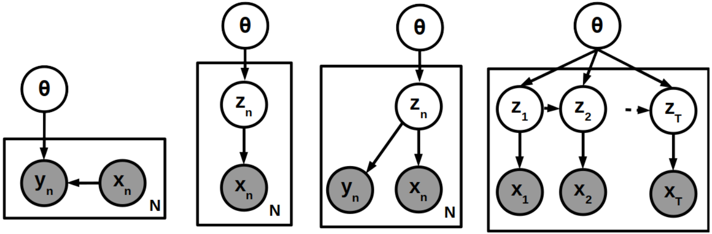

A Simple Supervised Learning Model

A Latent Variable Model for Unsupervised Learning

A Latent Variable Model for Supervised Learning

A Latent Variable Model for Sequential Data

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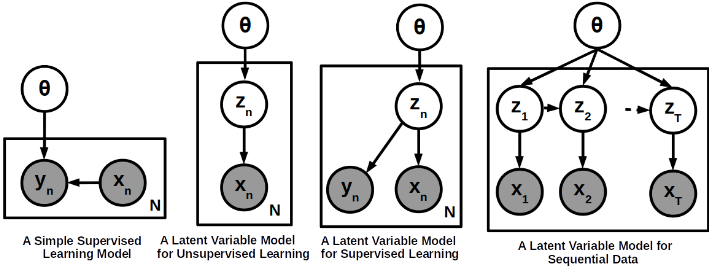

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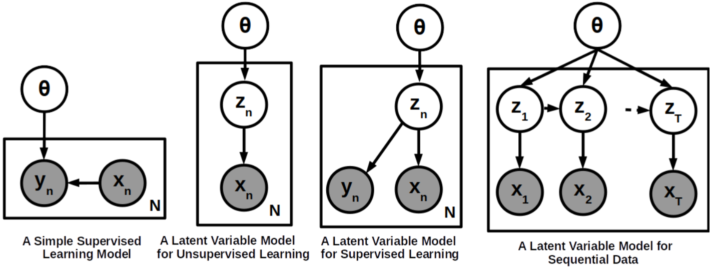

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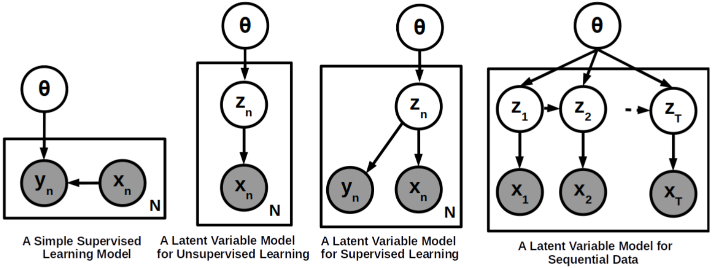

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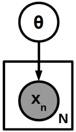
- The goal is to infer the unknowns of the model, given the observed data

Modeling Data Probabilistically

- Specification of probabilistic models requires two key ingredients: Likelihood and prior



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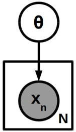

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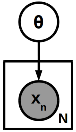

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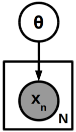

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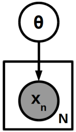
- Domain knowledge can help in the specification of the likelihood and the prior

Parameter Estimation/Inference in Probabilistic Models

- Perhaps the simplest way is to find θ that makes the observed data most likely or most probable



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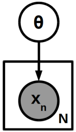

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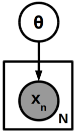

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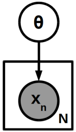

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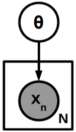

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- We will study both point estimation and Bayesian inference methods (and hybrids!)

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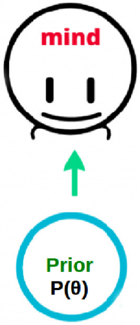
- Bayesian inference fits naturally into an "online" learning setting



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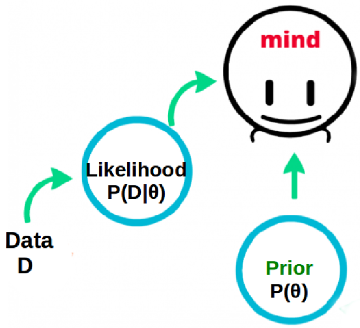

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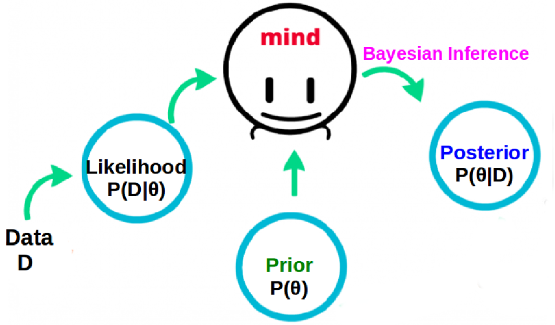

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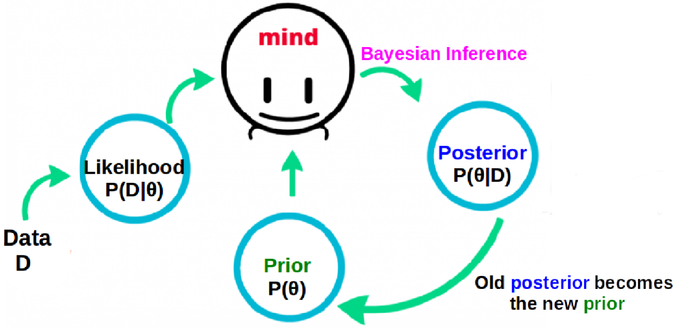

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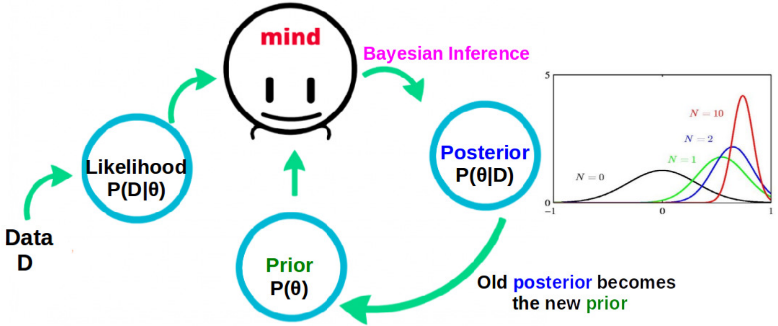

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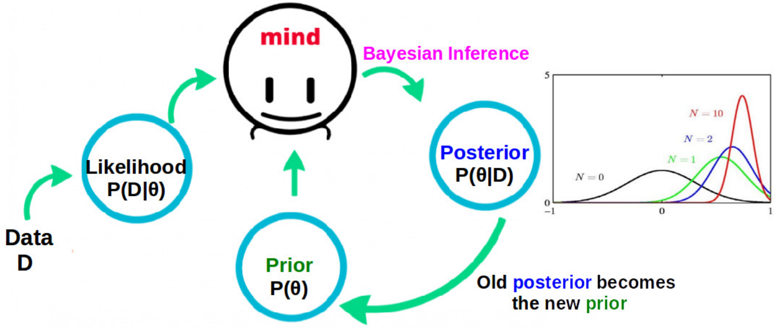
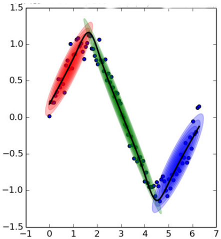


- Our belief about θ keeps getting updated as we see more and more data

Some Other Benefits of the Probabilistic Approach

Modular Construction of Complex Models

- Can easily construct combinations of multiple simple probabilistic models to learn complex patterns
- An example: Can perform nonlinear classification using a mixture of linear classifiers
  - It is a simple yet powerful combination of two models - one that performs clustering of the data and the other that learns a linear classifier within each cluster (both learned jointly)



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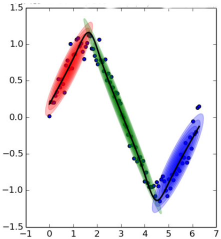
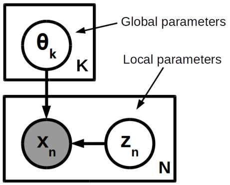


- More generally, these are called "mixture of experts" models

Generative Models

- Generative models of data can be naturally specified in a probabilistic framework



- Each data point $\boldsymbol{x}_n$ is associated with latent variables $\boldsymbol{z}_n$
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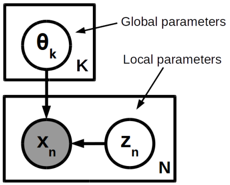

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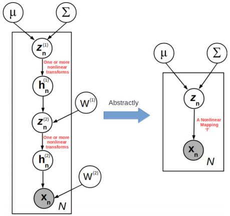
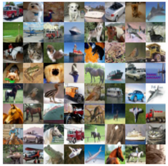
- Can also use the latent variables to infer missing data or relevance of each data point

(Deep) Generative Models

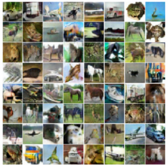
- Deep Generative Models for extremely popular nowadays (e.g., Variational Auto-encoders and Generative Adversarial Networks)



- Once learned, these models can also synthesize realistic looking "new" data from random $\boldsymbol{z}$'s



Real images (CIFAR-10)



Generated images

Averaging Over Posterior Distribution

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$$p(\boldsymbol{y}_* = 1|\boldsymbol{x}_*, \mathbf{X}, \boldsymbol{y}) = \int p(\boldsymbol{y}_* = 1|\boldsymbol{x}_*, \theta)p(\theta|\mathbf{X}, \boldsymbol{y})d\theta$$

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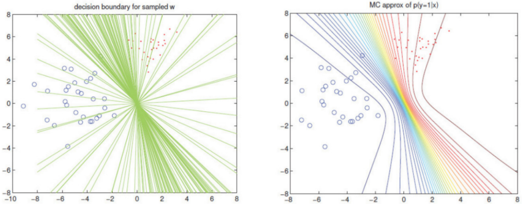
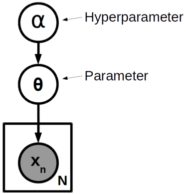
- Averaging leads to more robust predictions (and prevents overfitting)



Hyperparameter Estimation

- Every model invariably has certain hyperparameters, e.g., regularization hyperparater in a linear regression model, or kernel hyperparameters in nonlinear regression of kernel SVM, etc.



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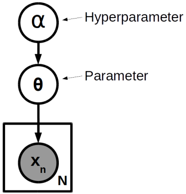

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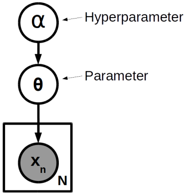

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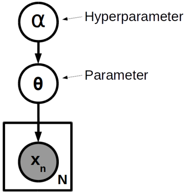

- The probabilistic approach enables learning the hyperparam. from data (without cross-validation)
    - Can put priors on the hyperparameters and infer the posterior distribution
    - Can do point estimation for hyperparameters by maximizing the marginal likelihood

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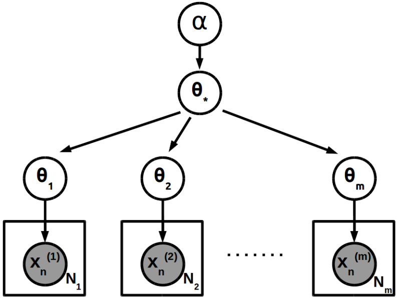
Multitask and Transfer Learning

- Allows joint learning across multiple data sets (known as multitask learning or transfer learning)



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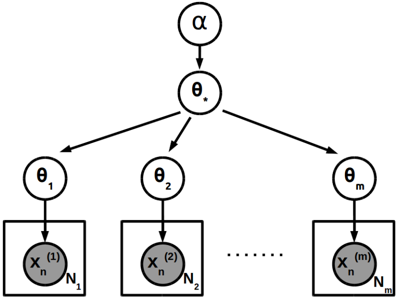

- Enables different but related models to "share statistical strength"

Model Comparison

- Suppose we have a number of models to choose from
- Let's compute the posterior probability of each candidate model, again using Bayes rule

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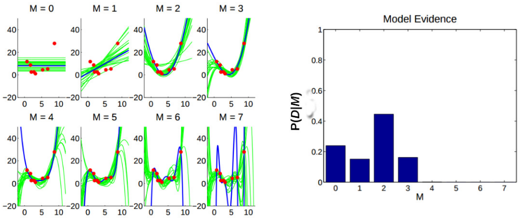

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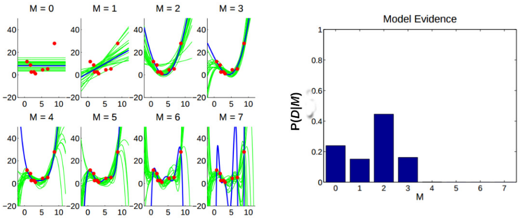
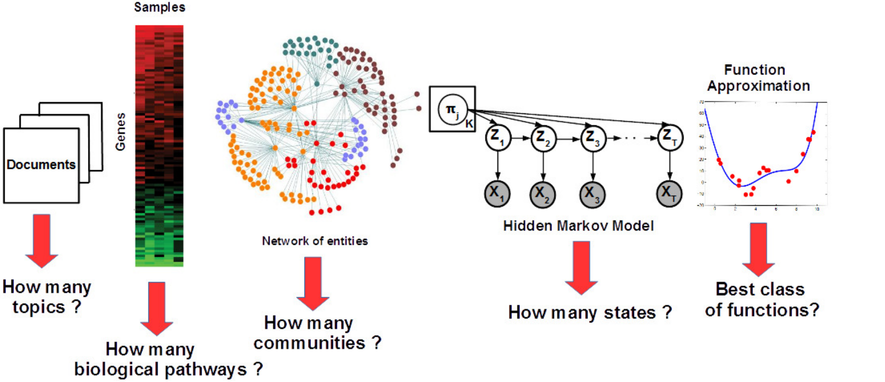


- It doesn't require a cross-validation set (can be done even for unsupervised learning problems)

Nonparametric Bayesian Modeling

- **Nonparametric Bayesian Modeling:** A principled way to learn "right" model size/complexity



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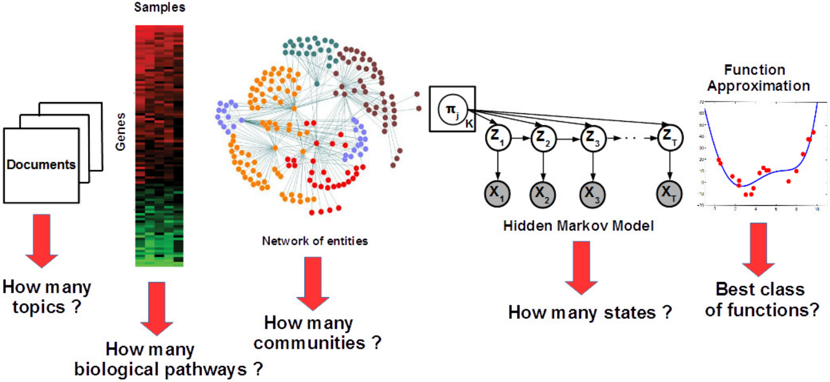

- The model size can grow with data (especially desirable for online learning settings)

Tentative Outline

- Basics of probabilistic modeling and inference
  - Common probability distributions
  - Basic point estimation (MLE and MAP)
- Bayesian inference (simple and not-so-simple cases)
- Probabilistic models for regression and classification
- Probabilistic Graphical Models
- Gaussian Processes (probabilistic modeling meets kernels)
- Latent Variable Models (for i.i.d., sequential, and relational data)
- Approximate Bayesian inference (EM, variational inference, sampling, etc)
- Nonparametric Bayesian methods
- Recent Advances, e.g., deep generative models, black-box inference, etc