CS201 Projects

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1 Proof Techniques

Project 1: **Induction in greater depth:** For reference see the book **Handbook of Mathematical Induction: Theory and Applications** (available at iitk central library). Study the chapter "Variants of Finite Mathematical Induction". Cover the following topics: weak induction, Strong mathematical induction, Downward induction. Alternative forms of mathematical induction, Double induction, Fermats method of infinite descent and Structural induction. Give a few examples on each form of induction.

Study the chapter "Inductive Techniques Applied to the Infinite". Cover the topics like well-ordered sets, Transfinite induction, Cardinals,Ordinals, Axiom of choice and its equivalent forms.

2 Countability

Project 1: **Topics in Countability:** Study Hilbert's paradox of the great hotel, Proof of Cantor-Schroder-Bernstein Theorem, Proof of uncounability of Cantors set. Know basics of ordinal number, transfinite induction, axiom of choice. For reference see Wikipedia.

3 Number Theory

Project 1: **A Few Number-theoretic Functions:** Study the following functions. For a given integer n, 1. $\tau(n)$, the number of positive divisors of n, 2. $\sigma(n)$, the sum of positive divisors of n, 3. The greatest integer function and 4. $\phi(n)$, the number of positive integers not exceeding n that are relatively prime to n.

For reference see Chapter 6-7 of **Elementary Number Theory** by David M. Burton.

Project 2: **Linear Diophantine Equations and Linear Congruences:** Solving Linear Diophantine Equations using Extended GCD algorithm and Simple

Finite Continued Fractions, Linear Congruences and the Chinese Remaindering Theorem.

For reference see following sections of **Elementary Number Theory** by David M. Burton:
1. For Solving Linear Diophantine Equations using Extended GCD algorithm see Chapter 2.5.
2. For Solving Linear Diophantine Equations using Simple Finite Continued Fractions see Chapter 15.2.
3. For Linear Congruences and the Chinese Remaindering Theorem see Chapter 4.4.

Project 3: The Quadratic Reciprocity Law: Definition of Quadratic residue, Euler's Criterion for testing quadratic residue, The legendre Symbol and its Properties, Quadratic Reciprocity Law and its some consequences.

For reference see Chapter 9 of **Elementary Number Theory** by David M. Burton.

4 Combinatorics

Project 1: Enumeration under group action: For reference see Chapter 15 of **Combinatorics: Topics, Techniques, Algorithms** by Peter Cameron.

Project 2: Linear Algebra methods in combinatorics: Study First two sections of Chapter 1, For Basic of Linear Algebra see Chapter 2 (if someone is confident in Linear algebra, he/she may skip) and Chapter 4 of **Linear algebra methods in combinatorics with applications to geometry and computer science** by László Babai.

Project 3: Partitions and introduction to Young Tableaux: Study chapter 3, 4 and 5 of **A combinatorial miscellany** available at `www-math.mit.edu/~rstan/papers/comb.pdf`

Project 4: Catalan Number: Study Chapter 1 and following combinatorial problems with solutions using Catalan Number from Chapter 2: Problem 1, 3, 4, 24 and 61 from **Catalan Numbers** by R. Stanley.

Project 5: Latin Squares and SDR: Study Chapter 6 of **Combinatorics: Topics, Techniques, Algorithms** by Peter Cameron.

Project 6: A few Theorems in Extremal Combinatorics and Extremal Graph Theory: Study Chapter 7 of **Combinatorics: Topics, Techniques, Algorithms** by Peter Cameron for Extremal Combinatorics. Study Mantel's Theorem and Túran's Theorem from Lecture 1 of `https://www.dpmms.cam.ac.uk/~dc340/Extremal-course.html`

Project 7: Art of finding combinatorial proofs: Study Chapter 1 (Fibonacci identities) and chapter 5 (Binomial identities) **Proofs that Really Count: The Art of Combinatorial Proof**

Project 8: **Introduction to Matroids:** Definitions of matroid, independent set, bases, rank, circuit. Examples of matroid from linear algebra and graph theory. Connection with greedy algorithms.

Read Lecture 3 from **The Design and Analysis of Algorithms** By Dexter C. Kozen

5 Graph Theory

Project 1: **Ramsey Numbers:** For reference see Chapter 11 of **Intrduction to Graph Theory** by Gary Chartrand and Ping Zhang.

Project 2: **Cayley's Formula and Kirchoff's Matrix-tree Theorem:** Study four different proofs of the Cayley's formula for the number of trees from Chapter 26 of **Proofs from THE BOOK** by M. Aigner and G.Ziegler. For Kirchoff's Matrix-tree Theorem see `http://www.math.ku.edu/~jmartin/mc2004/graph1.pdf`.

Project 3: **Art Gallery Theorem:**Study `http://cs.smith.edu/~orourke/books/ArtGalleryTheorems/Art_Gallery_Chapter_1.pdf` Chapter 39 of the "How to Guard a museum" from **Proofs from THE BOOK** by M. Aigner and G.Ziegler also has a nice exposition on this problem.

6 Discrete Geometry

Project 1: **Sylvester-Gallai Theorem and Crossing Lemma:** Study Section 1 containing The Sylvester-Gallai Theorem and Section 2 (before Section 2.1) containing The Crossing Lemma from `www.math.kit.edu/iag6/lehre/combplane2013s/media/lecture_notes.pdf`.

Project 2: **Pick's Theorem: Computing the Continuous Discretely:**Study from `www.geometer.org/mathcircles/pick.pdf`. Also study Proof of Pick's Theorem using Euler's Theorem. Study the connection between Pick's Theorem and Farey Sequences from `http://www.maths.ed.ac.uk/~aar/papers/bruck.pdf`.

Project 3: **Combinatorial convexity:** Study Chapter 1 of **Lectures in Discrete Geometry** by Jiri Matousek.

7 Probability

Project 1: **Random Walks:** Two examples: 1. Gamblers ruin 2. Random walk on graph and page rank. Go to `http://courses.csail.mit.edu/6.042/spring14/mcs.pdf` and see Chapter 20, Page No: 841-855

Project 2: **Hashing:** Definition of hashing and its aim, Families of Universal Hash Function, Construction of 2-Universal Families of Hash Functions, Strongly 2-Universal Families of Hash Functions and its use in perfect hashing. For reference see Chapter 13, Page No: 321-328 of **Probability and Computing: Randomized Algorithms and Probabilistic Analysis** by Michael Mitzenmacher and Eli Upfal.

Project 3: **Lovász Local Lemma:** Statement of the lemma with proof, Study the following applications: 1. Vertex disjoint cycles in k-regular digraph, 2. Coloring of k-uniform family, 3. The k-SAT problem. For reference see Chapter 19 of **Extremal Combinatorics With applications in Computer Science** by Stasys Jukna.

Project 4: **Paradoxes in probability:** Monty Hall Problem, Three Prisoner's problem, Bertrands paradox (in geometric probability), Nontransitive dice paradox, Two envelope's problem or exchange paradox, Simpson's paradox. For reference see Wiki pages on each of these paradoxes.

8 Applications

Project 1: **Introduction to Cryptography:** Basic setting of cryptosystem, What is symmetric key cryptosystem and asymmetric key cryptosystem, Diffe-Hellman key exchange protocol with proper mathematical explanation, RSA cryptosystem: key generation, encryption and decryption with proper mathematical explanation, ElGamal cryptosystem: key generation, encryption and decryption with proper mathematical explanation, Explain mathematically why all the above protocols are secure. For reference search online.

Project 2: **Introduction to Error-correcting codes:** For reference see Chapter 17 up to Section 17.5 of **Combinatorics: Topics, Techniques, Algorithms** by Peter Cameron.

Project 3: **Introduction to Markov Chain and Hidden Markov Model:** Study the chapter "Markov Chains" from **Pearls of discrete mathematics**. For hidden markov model,see `http://di.ubi.pt/~jpaulo/competence/tutorials/hmm-tutorial-1.pdf` and read the article **An introduction to Hidden Markov Model** by Rabiner-Zheung.

9 Games and magic

Project 1: **Combinatorial games:** Study Chapters 0 and 1 of **Lessons in play: An introduction to combinatorial game theory** by M. Albert, R. Nowakowski, D. Wolfe. Study rules and strategies of the following games Tic-Tac-Toe, Nim, Hex.

Project 2: **Cops and Robber game on graphs:** Study the first chapter of **The Game of Cops and Robbers on Graphs** by A. Bonato and R. Nowakowski. In the chapter two, study till the proof of the theorem 2.3 "A graph is cop-win if and only if the graph is dismantlable". For a quick description of the game, see http://math.ucsd.edu/~fan/152/arch/coprob/.

Project 3: **Mathematical magics:** Study Chapter 1,2 and 3 of the following book http://press.princeton.edu/titles/9510.html. For De Bruijn card trick, see Problem 5 (de Bruijn Sequences and Universal cycles) of http://www.turgor.ru/lktg/2014/5/index.htm.

10 Algorithmic

Project 1: **Stable Marriage Problem:** Study first three chapters of **Stable Marriage and Its relation with other Combinatorial Problems** by Donald Knuth

11 Related topics

Project 1: **Introduction to Information Theory:** For reference see Chapter 16, 17 and 18 of **Pearls of Discrete Mathematics** by M. Erickson(available at IITK Library). Another elementary book covering the topics is "Diary of Information Theory" by A Rényi.