**Problem Statement**

- Discovering state space
- Discovering reward function
- Bring in spatio-temporal features
- Formulation of joint manifolds and random projections
- Path planning for visual servoing

**Abstract**

Problems in Robot motion planning
- High dimensional state space
- High dimensional sequential decision making
- High dimensional sensorimotor state space
- High computation cost
- Multiple domain receptor of a single scene

Solution
- Construction of low dimensional representation of robot’s state space
- All computations for robot’s decision making in reduced space

**Random Projections and Joint Manifolds**

- The set of all $K$-sparse signals are a non-linear union of $\binom{[N]}{K}$ dimensional subspaces
- The theory of compressed sensing states that every $K$-sparse signal can be recovered from just $M = O(K \log(N/K))$ measurements
- The basis matrix $\Phi_{MN}$ is a homomorphism, that is no two signals in $\mathbb{R}^N$ are mapped to same point in $\mathbb{R}^M$
- This mapping is ensured with probability 1 if $i$ has i.i.d entries and $M \geq 2K$, not guaranteed a stable embedding
- Random Projection theorem: Let $\mathcal{M}$ be a compact $K$-dimensional Riemannian submanifold of $\mathbb{R}^N$ having condition number $1/\varepsilon$, volume $V$ and a geodesic covering regularity $R$.
- Fix $0 < \varepsilon < 1$ and $0 < \rho < 1$. Let $\Phi$ be a random orthonormal projector from $\mathbb{R}^N$ to $\mathbb{R}^M$ with $M = O\left(K \log(NV R^{-1} \varepsilon^{-1}) \log(1/\rho)\right)$

If $M \leq N$, then with probability at least $1 - \rho$ the following statement holds: For every pair of points $x, y \in \mathcal{M}$,

$$\frac{1}{N} \left| \frac{\phi x - \phi y}{\| \phi x - \phi y \|} \right| \leq (1 + \varepsilon) \frac{1}{N} \left| \frac{x - y}{\| x - y \|} \right|$$

- Joint Manifolds:
  - Cameras: $J$.Dimension $N$. Total Dimension space: $JN$
  - Assumption Manifold alignment is present
  - $\mathcal{M}^* = \{ \varphi \in \mathcal{M} : \varphi = \psi(p_j), 2 \leq j \leq f \}$
  - $\mathcal{M} = \mathcal{M}^1 \times \mathcal{M}^2 \times \ldots \times \mathcal{M}^J$
  - Random Projections used in Joint manifold creation

**Grassberger –Procaccia algorithm**

- Suppose $X = \{x_1, x_2, \ldots, x_n\}$ is a finite dataset of underlying dimension $K$. Define

$$G_n(r) = \frac{1}{n(n-1)} \sum_{i=1}^{n-1} \|x_i - x_j\| < r$$

The scale-dependent correlation dimension of $X$ is defined as

$$D(r_1, r_2) = \frac{\log G_n(r_2) - \log G_n(r_1)}{\log r_2 - \log r_1}$$

**Manifold Learning using Random Projections**

- $M \leftarrow 1$
- $\Phi \leftarrow$ Random orthonormal projector of size $MN$
- While residual variance $\geq \gamma$ do
  - Run the GP algorithm on $\phi x$
  - Use intrinsic dimension ($K$) estimate to perform Isomap on $\phi x$
  - Calculate residual variance
  - $M \leftarrow M + 1$
  - Add one row to $\phi$
- End while
- Return $M$
- Return $\mathcal{K}$

**Path Planning for Visual Servoing**

- 4 main constraints
  - Target continuously within camera’s field of view
  - Avoiding visual occlusions of target by workspace’s obstacles or robot’s body
  - Avoiding collision with physical obstacles or self collision
  - Joint limits

**ST Isomap**

- Isomap and PCA don’t include time dependency, hence miss proximal data points
- Windowed MDS was able to get proximal data points but fails for corresponding spatially distal points which are equivalent phases in temporal process

**NuMax**

- Given a dataset $X \subset \mathbb{R}^N$, we aim to find a linear embedding $\mathcal{P} : \mathbb{R}^N \rightarrow \mathbb{R}^M, M \ll N$. Form the secant set $S(X) = \{v_1, v_2, \ldots, v_N\}$ and find a measurement matrix $\Psi \in \mathbb{R}^{MN \times M}$ that satisfies the RIP on secant set
- $P = \Psi^T \Psi \in \mathbb{S}^{NN \times MN}$, rank($P$) = $M$
- $\mathcal{A} : X \rightarrow \{v_i^T X v_i\}_{i=1}^{MN}$
- Instead of rank minimization, minimize nuclear norm, which for a positive semi-definite matrix is equal to its trace
- Minimize $\|P\|_*$
- Subject to $P = P^T$, $\|\mathcal{A}(P) - 1\|_\infty \leq \delta$
- $P^* = \mathcal{U} \mathcal{D} \mathcal{U}^T$, $\Psi = \mathcal{A}_M \mathcal{U}^T$

**References**


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