# Progressive Stochastic Search

## Application to solving sudoku

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# Sudoku

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# Sudoku: Problem Definition

A Sudoku square of order n consists of $n^4$ variables formed into a $n^2$ x $n^2$ grid such that:

1. Each row of cells contains the integers 1 through $n^2$ exactly once.
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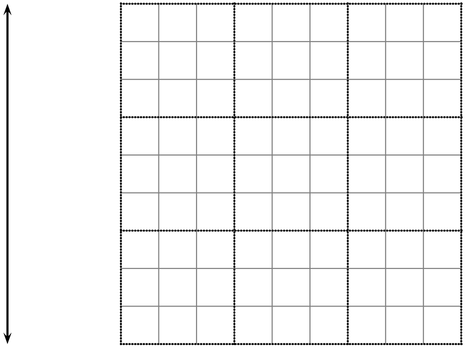
# A simple illustration of PSS



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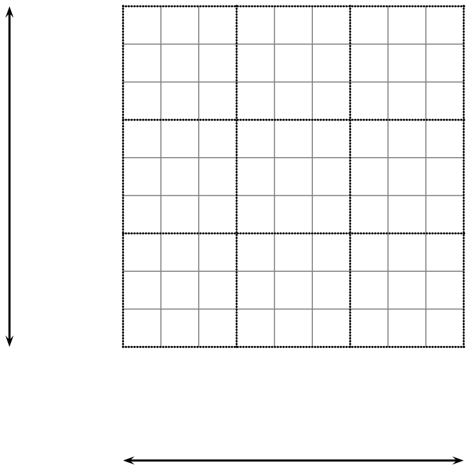

# Motivation

- All Sudoku puzzles are not logic-solvable; many involve guesswork
- Proved to be an NP-complete problem: a polynomially bound algorithm for solving all problem instances is not possible
- It is claimed that even for order-3 Sudoku puzzles, there are 6,670,903,752,021,072,936,960 valid arrangements (!)
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# Stochastic Search

- Typical SS algorithms first generate a complete initial variable assignment and repair the assignment by heuristic local search with reference to a cost function until a solution is found.
  - Problem: Might get stuck on a plateau or in a local optima
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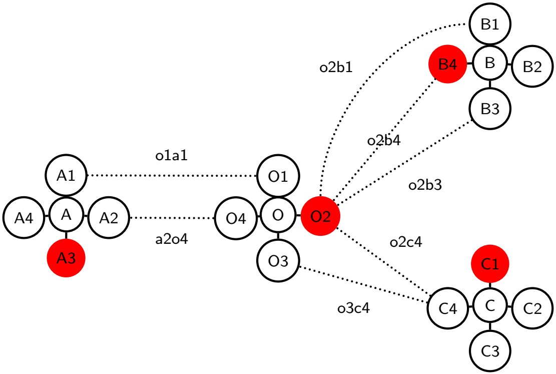

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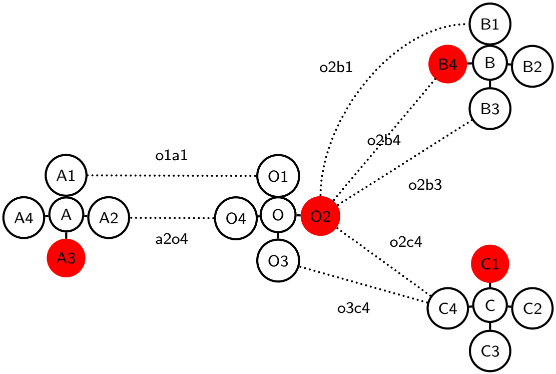
**Step 1**

Pick up the head cluster O of queueQ.



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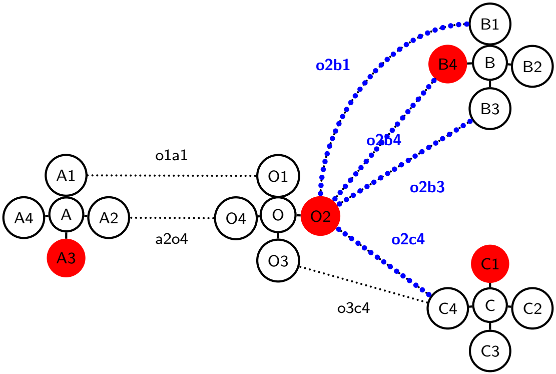
**Step 2**

$conflicts_{O2} = o2b1 + o2b3 + o2b4 + o2c4 \neq 0$



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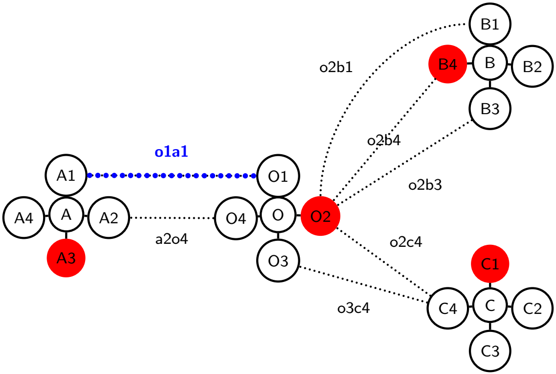
## Step 3

$conflicts_{O1} = o1a1$



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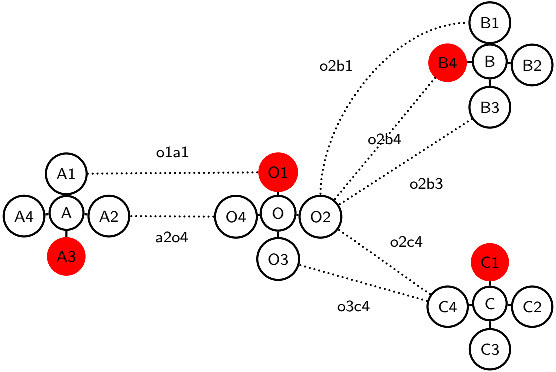
## Step 4

$value_O := O_1$



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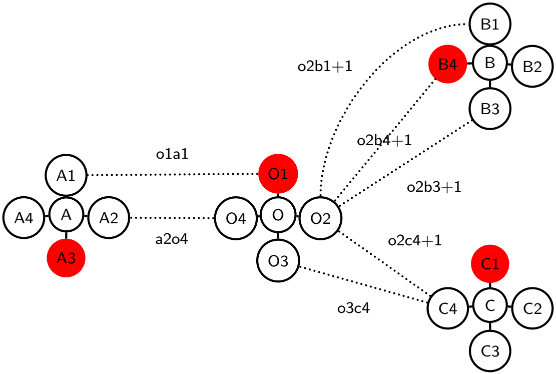
**Step 5**

$o2b1 := o2b1 + 1$ $o2b3 := o2b3 + 1$
$o2b4 := o2b4 + 1$ $o2c4 := o2c4 + 1$



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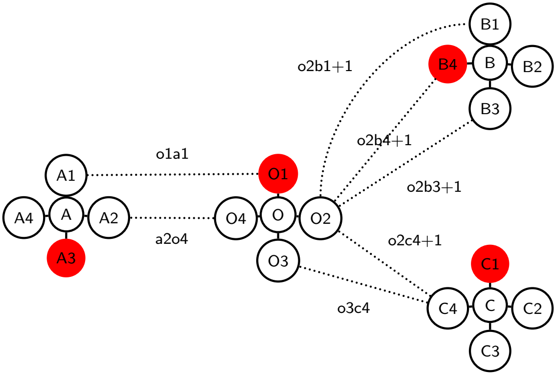
## Step 7

Append $A_1$ to the queue



# Progressive Stochastic Search

- Maintains a list of variables, which dictate the sequence of variables to be repaired
- When a variable is designated to be repaired in PSS, it always chooses a new value even if its original value gives a better cost
- Search paths slightly marked by worsening at every point on the paths
- Intuitively, it's driven by a "force" so that the search is able to "rush through" the local minima and plateaus

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# Modelling Sudoku as a CSP

- The Sudoku puzzle will be modelled as a grid of $n^4$ cells - each of which represents an integer variable which initially will have a domain of 1 through $n^2$
- Constraints can be added in the form of "alldifferent" constraints and using the pre-filled cells in the grid
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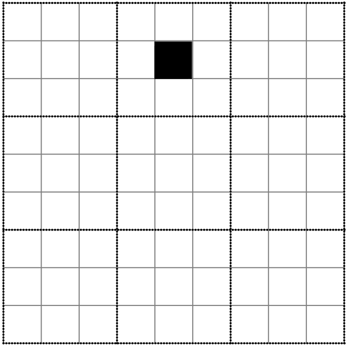

$Domain = Domain - v$

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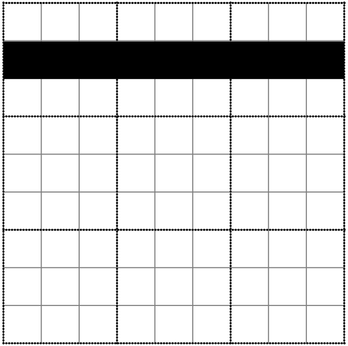

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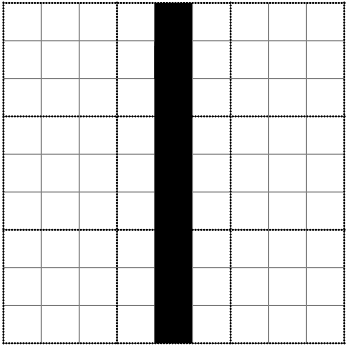

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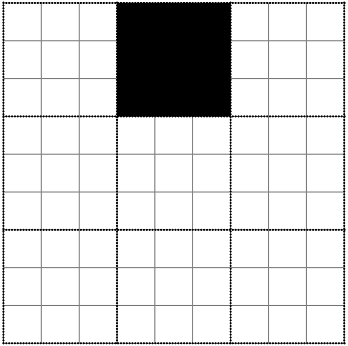

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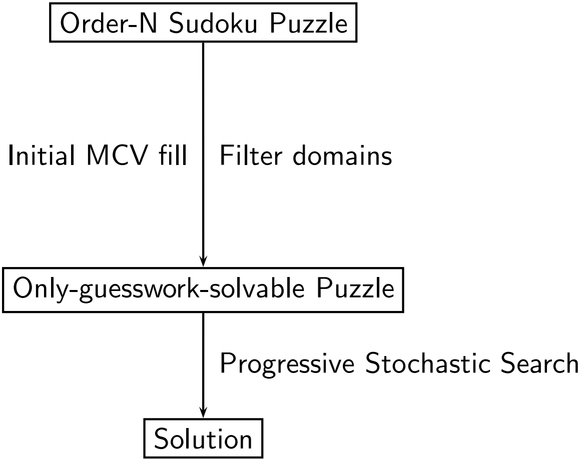
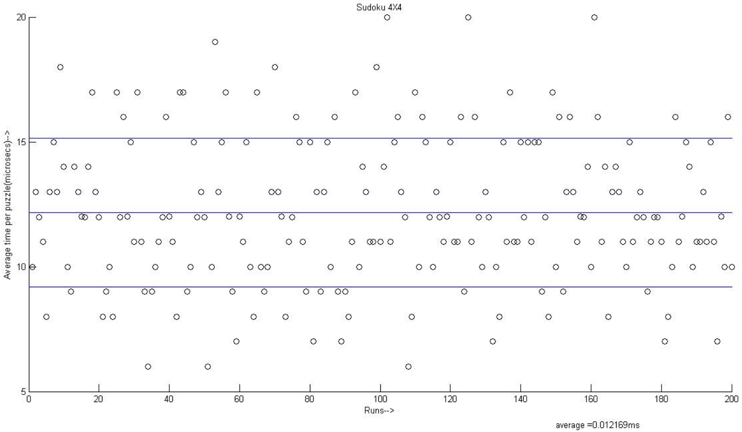
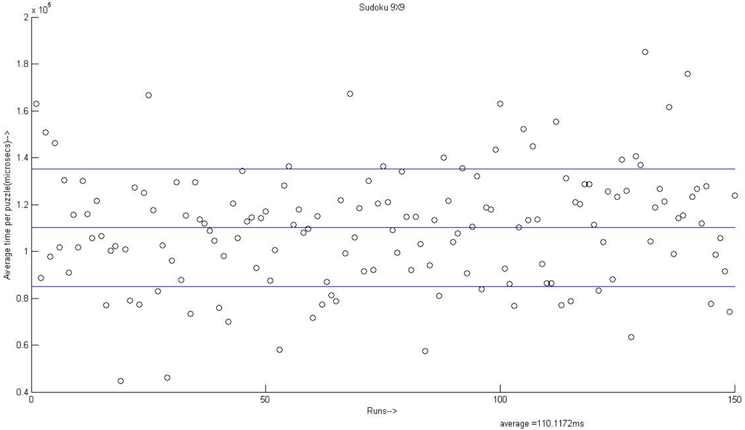
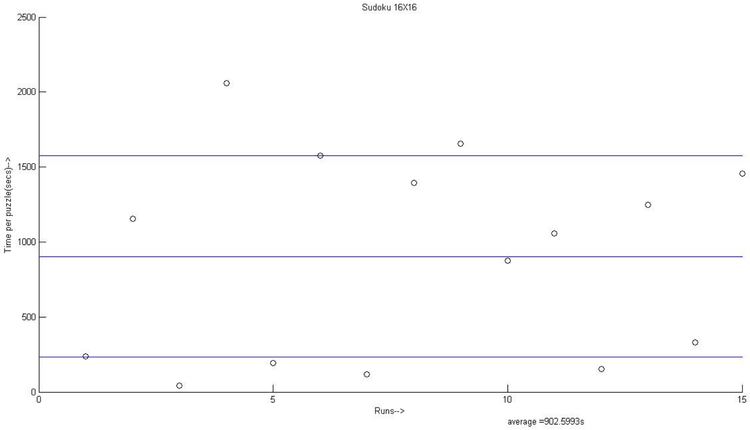



Sudoku 4X4
Average time per puzzle(microsecs)-->
Runs-->
20
15
10
5
0
20
40
60
80
100
120
140
160
180
200
average =0.012169ms


Sudoku 9X9
x 10^5
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
Average time per puzzle(microsecs)-->
0
50
100
150
Runs-->
average =110.1172ms


Sudoku 16X16
2500
2000
1500
1000
500
0
Time per puzzle(secs)-->
0
5
10
15
Runs-->
average =902.5993s

# Acknowledgements


- Bryan Chi-ho Lam and Ho-fung Leung, 2003. Progressive Stochastic Search for Solving Constraint Satisfaction Problems. Proceedings of the 15th IEEE International Conference on Tools with Artificial Intelligence (ICTAI03) 1082-3409/03.
- Rhydian Lewis, 2007. On the Combination of Constraint Programming and Stochastic Search. The Sudoku CaseProceedings of the 4th international conference on Hybrid metaheuristics.
- Helmut Simonis, 2005, Sudoku as a Constraint Problem, IC-Parc, Imperial College, London