Proceedings of the 2010 IEEE
International Conference on Robotics and Biomimetics
December 14-18, 2010, Tianjin, China

An Online Motion Planning Algorithm for a 7DOF Redundant Manipulator

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***Abstract*— For a 7 degree of freedom (7DOF) manipulator which has redundant DOF, traditional numerical algorithm for the inverse kinematic model just provides only one possible solution for desired position and orientation of end-effector which can not fully use the redundancy feature of the 7DOF manipulator. This paper proposes an online motion planning algorithm for a 7DOF manipulator with an analytical inverse kinematic model. This algorithm uses a new constraint named Elbow Angle besides traditional position and orientation matrix constraint to solve the inverse kinematic model. By Using this new constraint, an analytical solution of the inverse kinematic model for this 7DOF manipulator is deduced and the Elbow Angle is autonomously decided online by the environment information. The simulation results demonstrate the high accuracy, fast computation and valuable application of this algorithm.**

I. INTRODUCTION

Robotics is one of the hottest research areas that draw much attention of scientists and engineers. Robot has a broad application or potential application in almost every area in human life such like manufacture automation, unknown area exploration, human search and rescue in catastrophe event and handicapped people assistance [1]. In a robot system, manipulator is one of the most important compositions of a robot which can achieve many humanoid function, for example, object grasping and delivering. There are many kinds of manipulator. One way to classify it is based on the amount of degree of freedom (DOF) a manipulator maintains. One kind of manipulator is classified as non-redundant manipulator which has 6DOF or less such like PUMA manipulator [2] and Stanford manipulator [3] which both has 6DOF. Another kind of manipulator is called redundant manipulator which has more than 6DOF such like the LWA3 manipulator manufactured by Schunk Company which has 7DOF. Because the redundant manipulator has more DOF than necessary for position and orientation computation, for a specific position and orientation of end effector, multiple solution exist. For this reason, the redundant manipulator has much more advantages in joint limit constraint avoidance [4], obstacle avoidance [5] and singularity avoidance [6] compared to the non-redundant manipulator.

Traditional work on redundant manipulator ether can not use the redundancy of the manipulator or just can make motion planning offline. However some tasks require the robot to work in some unknown place with obstacles such like mission of search and rescue in earthquake area. During this type of task the manipulator on the mobile robot is often tele-controlled by a tele-operator with a joystick. The tele-operator sends six-dimension position and orientation command or velocity command to the manipulator through the joystick. However, this command just controls the position and orientation of end-effector of the manipulator and consequently it can not decide the position of other part of the manipulator. This sometimes may cause big trouble because in some cases when the tele-operator controls the end-effector to one specific position and orientation however in the same time some links of the manipulator hit the obstacle. This situation may probably lead to a serious damage to the manipulator. To avoid this, this paper deduces a motion planning algorithm of the manipulator.

This paper will be organized as below. The basic description of this 7DOF redundant manipulator and the forward kinematic model will be introduced in Section II. Secondly, the online motion planning algorithm will be deduced in Section III. In Section IV, a simulation result will be given. Finally, the conclusion and future work will be demonstrated in Section V.

II. FORWARD KINEMATIC OF 7DOF REDUNDANT MANIPULATOR

The 7DOF redundant manipulator is shown in Fig. 1. We can see that all the joints of this manipulator is revolve joint and any two axis of each joint is perpendicular to each other as well as cross in one point. Besides, the gripper is attached to the 7th link. To build the forward kinematic model, the coordinate of every link is defined base on a classical Denavit-Hartenberg model [7].

The base coordinate frame of the manipulator is defined as Σ_0. Then, each joint $i(i = 1, 2...6)$ is defined as Σ_i based on the Denavit-Hartenberg rules (DH rules). For the coordinate frame Σ_i, the origin is placed on the joint axis $i + 1$, and the $z-$axis is aligned with the joint axis. The $x-$axis is perpendicular to the common normal of the joint axes i and $i+1$. The $y-$axis is then decided to form a right-hand coordinate system. The origin of coordinate system Σ_7 is defined on the tip of the gripper based on DH rules. Which should be mentioned is that there are may be exist different definition of the coordinate of each joint base on the DH rule,

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978-1-4244-9318-0/10/$26.00

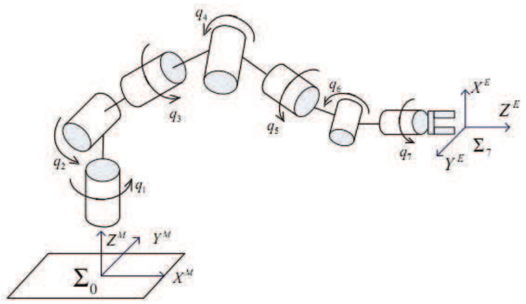

Fig. 1. 7DOF Redundant Manipulator

TABLE I
DH MODEL OF 7DOF MANIPULATOR

i	$a_i(mm)$	α_i	$d_i(mm)$	q_i
1	0	-90^o	300	*var*
2	0	90^o	0	*var*
3	0	-90^o	328	*var*
4	0	90^o	0	*var*
5	0	-90^o	276.5	*var*
6	0	90^o	0	*var*
7	0	0	380	*var*

and different definition leads to different DH parameters.

Based on the DH rules, the DH parameters of the manipulator are showed in table 1.

In TABLE I, a_i , α_i , d_i , q_i are all DH model parameters, *var* means q_i is a variable. Used these DH parameters, the transformation matrix of each coordinate frame can be got:

$$A_1 = \begin{pmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
$$A_3 = \begin{pmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, A_4 = \begin{pmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
$$A_5 = \begin{pmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}, A_6 = \begin{pmatrix} C_6 & 0 & S_6 & 0 \\ S_6 & 0 & -C_6 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
$$A_7 = \begin{pmatrix} C_7 & 0 & -S_7 & 0 \\ S_7 & 0 & C_7 & 0 \\ 0 & 0 & 1 & d_7 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

In (1), S_i ,C_i means $\sin(q_i)$, $\cos(q_i)$ separately. A_i is the transformation matrix from coordinate Σ_{i-1} to Σ_i . By multiplying these transformation matrixes together, the transformation matrix from end-effector to the base coordinate can be written as:

$$H_7^0 = \prod A_i \tag{2}$$

Using formation (2) we can get the position and orientation of the end-effector with specific joint angles $Q = \begin{pmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 \end{pmatrix}^T$. We also call this matrix H_7^0 Destination Matrix.

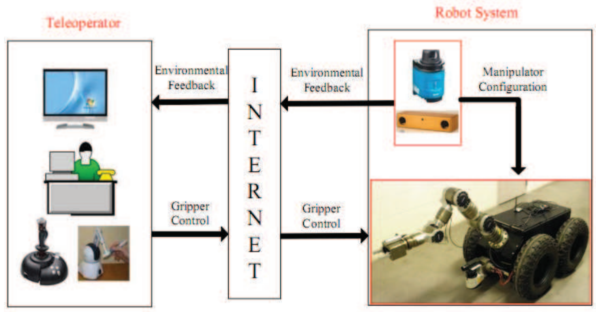

Fig. 2. Architecture of the Mobile Robot System

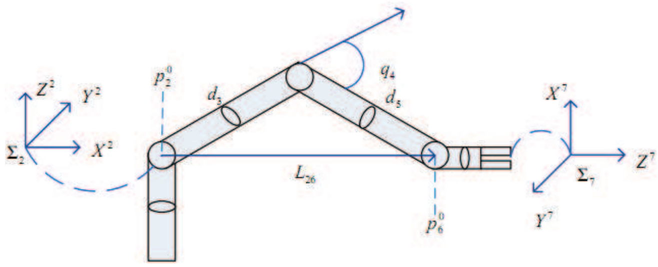

Fig. 3. Chart for solving q_4

III. ONLINE MOTION PLANNING OF 7DOF REDUNDANT MANIPULATOR

A. Architecture of the Online Motion Planning Algorithm

The architecture overview of our sensor based online motion planning algorithm is provided in Fig.2. It is assumed that the redundant manipulator Online Motion Planning controller receives the six dimensional position and orientation command of the end-effector from the tele-operator and at the same time the sensors send the environment information to the controller. With the fusion of this information, the controller will find the best motion of the redundant manipulator based on different criterion. On the following, an obstacle avoidance based motion planning algorithm will be deduced.

B. Solving Joint Angle q_4

To the 7DOF redundant manipulator discussed in this paper, for a specific Destination Matrix H_7^0 of it, the origin of coordinate system Σ_6 can be verified solely. Because the joint angle value q_7 just change the orientation relationship of coordinate system Σ_6 and coordinate system Σ_7 while it have no affect on the position relationship of the two coordinates systems. Then the origin of the coordinate system Σ_6 can be calculated (Fig.3.):

$$p_6^0 = H_7^0 \begin{pmatrix} 0 \\ 0 \\ -d_7 \\ 0 \end{pmatrix} = \begin{pmatrix} p_{6x} \\ p_{6y} \\ p_{6z} \\ 0 \end{pmatrix} \tag{3}$$

The same case, the joint angle q_1 will not affect the position relationship of coordinate system Σ_2 and coordinate system Σ_0 which means that the origin of coordinate system Σ_2 is $p_2^0 = \begin{pmatrix} 0 & 0 & d_1 \end{pmatrix}^T$. Based on the analysis above, it is obvious that when the Destination Matrix is fixed, the position of p_6^0 and p_2^0 are also verified. Then the distance of these two points can be calculated:

$$L_{26} = \left\| p_6^0 - p_2^0 \right\| \tag{4}$$

Based on the physic structure of the manipulator, it is easy to find that joint 2, joint 4 and joint 6 form a triangle. The distance from joint 2 to joint 4 and from joint 4 to joint 6 are constant which are d_3 and d_5 . Since the length of the three sides has been identified, q_4 is calculated with cosine theorem:

$$q_4 = \pi - \arccos\left(\frac{d_3^2 + d_5^2 - L_{26}^2}{2d_3^2 d_5^2}\right) \tag{5}$$

C. Definition of a Extra Constraint

This 7DOF redundant manipulator has an interesting and useful feature that the rotation axis of the first three joints perpendicular to each other and cross in one point. Korein has proved this type of structure is equal to a spherical joint and it can rotate through axis simultaneously [8].

As showed in Fig.4, the structure composed by the first three joints is defined as Shoulder (S) like human arm and similarly the structure composed by the last three joints is also defined as Wrist (W). Since the Shoulder and Wrist are both equal to spherical joints, the Elbow (E) can rotate around the virtual axis *SW* . The path of the Elbow is a circular. In order to calculator the position of Elbow, a mathematical expression of this circular is deduced below. The coordinate system of Elbow rotation plane is defined as $\begin{pmatrix} \vec{x_E} & \vec{y_E} & \vec{z_E} & O_E \end{pmatrix}$, in which O_E is the origin and $\vec{x_E}, \vec{y_E}, \vec{z_E}$ are three axis respectively. Based on base coordinate system Σ_0 :

$$\begin{aligned} \vec{z_E} &= \frac{p_6^0 - p_2^0}{\|p_6^0 - p_2^0\|}, \vec{x_E} = \frac{\vec{z_0} \times \vec{z_E}}{\|\vec{z_0} \times \vec{z_E}\|}, \\ \vec{y_E} &= \frac{\vec{z_E} \times \vec{x_E}}{\|\vec{z_E} \times \vec{x_E}\|}, O_E = \cos(\beta) * d_3 * z_E \end{aligned} \tag{6}$$

in which, $z_0 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T, \beta = \arccos\left(\frac{L_{26}^2 + d_3^2 - d_5^2}{2L_{26}d_5}\right)$ and radius of this circular $R = d\sin(\beta)$.

After the definition of rotation plane coordinator system, an angle that express the Elbow rotating through $\vec{z_E}$ can be defined as Elbow Angle α_E

$$\alpha_E = \angle E O_E \vec{x_E} \tag{7}$$

Based on the analysis in last chapter, it can be found that the ideal range of Elbow Angle α_E is from 0 to 2π. For any specific value of Destination Matrix H_7^0 and Elbow Angle α_E , the poison of Elbow is fixed and can be calculated as:

$$E(\alpha_E) = O_E + R(\cos(\alpha_E)\vec{x_E} + \sin(\alpha_E)\vec{y_E}) \tag{8}$$

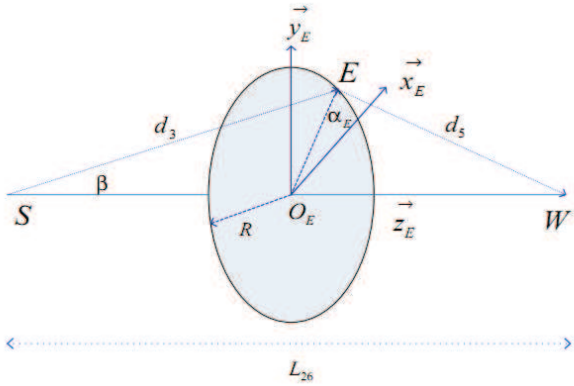

Fig. 4. Definition of Elbow Angle

D. Calculation of Elbow Angle α_E with Plane Constrain

It is assumed that the 3D shape of the obstacles has been detected in time by sensors such like lasers and cameras. Then these obstacles can be modeled as a plane. One side of this plane is "safe" without obstacles and another side of the plane is "dangerous" with obstacles. We call this plane as "safe plane". There are many way to create this plane, one easy way used here is that build the plane with three points of the obstacle. For example, the three points can be the points that are nearest to the base coordinate origin of manipulator.

Assume the three points are $\begin{pmatrix} p_{1x} & p_{1y} & p_{1z} \end{pmatrix}$, $\begin{pmatrix} p_{2x} & p_{2y} & p_{2z} \end{pmatrix}$, $\begin{pmatrix} p_{3x} & p_{3y} & p_{3z} \end{pmatrix}$. Then the plane is calculated as: $ax + by + cz + d = 0$.In which, $a = \frac{-d}{D}\begin{vmatrix} 1 & p_{1y} & p_{1z} \\ 1 & p_{2y} & p_{2z} \\ 1 & p_{3y} & p_{3z} \end{vmatrix}$, $b = \frac{-d}{D}\begin{vmatrix} p_{1x} & 1 & p_{1z} \\ p_{2x} & 1 & p_{2z} \\ p_{3x} & 1 & p_{3z} \end{vmatrix}$,$c = \frac{-d}{D}\begin{vmatrix} p_{1x} & p_{1y} & 1 \\ p_{2x} & p_{2y} & 1 \\ p_{3x} & p_{3y} & 1 \end{vmatrix}$, $d = \begin{vmatrix} p_{1x} & p_{1y} & p_{1z} \\ p_{2x} & p_{2y} & p_{2z} \\ p_{3x} & p_{3y} & p_{3z} \end{vmatrix}$. Because the origin of the base coordinate is always in the "safe" side of the plane. We want the Elbow also in the same side, which means:

$$(\begin{pmatrix} 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} a \\ b \\ c \end{pmatrix} + d) * (E(\alpha_E) * \begin{pmatrix} a \\ b \\ c \end{pmatrix} + d) > 0 \tag{9}$$

From (8) and (9) we can rewrite the inequality equation as:

$$A\cos(\alpha_E) + B\sin(\alpha_E) + C > 0 \tag{10}$$

To solve this inequality equation, a triangle transform is used:

$$\begin{aligned} &A\cos(\alpha_E) + B\sin(\alpha_E) + C = r\cos(\alpha_E - t) + C, \\ &r = \sqrt{A^2 + B^2}, t = atan2(B, A) \end{aligned} \tag{11}$$

By solving the inequality $cos(\alpha_E - t) + C > 0$. The result "safe" range of Elbow Angle is:

$$\alpha_{E} \subset (\alpha_{E\_S}, \alpha_{E\_B}) = (-[\arccos(-\frac{C}{r})+t], [\arccos(-\frac{C}{r})+t]) \quad (12)$$

If the obstacle are modeled as multiply plane $P_1, P_2, ..., P_n$, the "safe" range is $\alpha_{E}^{P_1} \bigcap \alpha_{E}^{P_2}, ... \bigcap \alpha_{E}^{P_n}$. It should be mentioned that with the data get from the sensors, the so-called "safe plane" is constructed in real-time and the Elbow Angle α_{E} is dynamically calculated.

E. Solving q_1, q_2, q_3 and q_5, q_6, q_7

In this kind of 7DOF redundant manipulator, the values of joint angle do not affect the position of S, E, W which means that the plane composed by S, E, W is solely verified by q_1, q_2, q_3 .

As shown in Fig.5, we define the origin position and orientation of S, E, W plane as Σ_{Origin} destination position and orientation of this plane is Σ_{Desti} . The origin position and orientation is when q_1, q_2, q_3 are equal to 0 and the destination position and orientation is when $W = p_6^0, E = E(\alpha_{E})$.

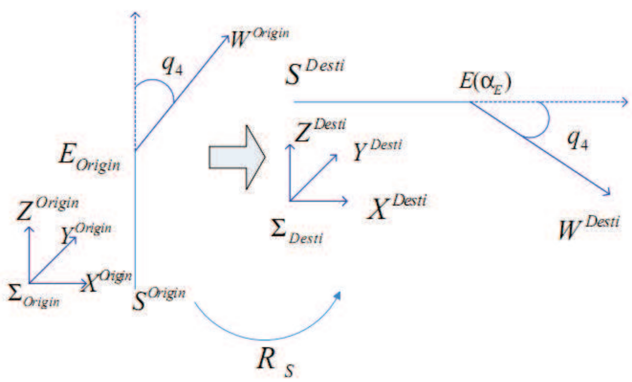

Fig. 5. Rotation of SEW Plane

Σ_O is short for Σ_{Origin}, and Σ_D is short forΣ_{Desti} ,then $\Sigma_O$is defined as:

$$\begin{aligned} Z^O &= \frac{E^O - S^O}{\|E^O - S^O\|} \\ X^O &= \frac{(W^O - S^O) - ((W^O - S^O) \cdot Z^O)Z^O}{\|(W^O - S^O) - ((W^O - S^O) \cdot Z^O)Z^O\|} \\ Y^O &= Z^O \times X^O \end{aligned} \quad (13)$$

Similarly, Σ_D is defined as:

$$\begin{aligned} Z^D &= \frac{E^D - S^D}{\|E^D - S^D\|} \\ X^D &= \frac{(W^D - S^D) - ((W^D - S^D) \cdot Z^D)Z^D}{\|(W^D - S^D) - ((W^D - S^D) \cdot Z^D)Z^D\|} \\ Y^D &= Z^D \times X^D \end{aligned} \quad (14)$$

Then the rotation matrix of Shoulder from Σ_O to Σ_D can be calculated as:

$$R_s = \begin{pmatrix} X^D & Y^D & Z^D \end{pmatrix} \begin{pmatrix} X^O & Y^O & Z^O \end{pmatrix}^T \quad (15)$$

There is another expression of this rotation matrix. Because q_1, q_2, q_3 are the ZYZ Euler angle of spherical joint S,R_s can be rewritten as:

$$R_s(q_1, q_2, q_3) = \begin{pmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{pmatrix} \begin{pmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (16)$$

We can see that R_s and $R_s(q_1, q_2, q_3)$ are the same rotation matrix, so let $R_s(q_1, q_2, q_3) = R_s$, then q_1, q_2, q_3 can be calculated. The solving for q_5, q_6, q_7 is extremely similar with q_1, q_2, q_3. Destination matrix is rewritten as $H_7^0 = \begin{pmatrix} R_7^0 & p_7^0 \\ 0 & 1 \end{pmatrix}$, in which R_7^0 is the destination rotation matrix. It is easy to get $R_w = (R_s R(q_4))^{-1} R_7^0$, in which $R(q_4) = \begin{pmatrix} C_4 & 0 & S_4 \\ 0 & 1 & 0 \\ -S_4 & 0 & C_4 \end{pmatrix}$.

q_5, q_6, q_7 are also the ZYZ Euler angle of spherical joint W and R_w can also be rewritten as:

$$R_w(q_5, q_6, q_7) = \begin{pmatrix} C_5 & -S_5 & 0 \\ S_5 & C_5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_6 & 0 & S_6 \\ 0 & 1 & 0 \\ -S_6 & 0 & C_6 \end{pmatrix} \begin{pmatrix} C_7 & -S_7 & 0 \\ S_7 & C_7 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (17)$$

R_w and $R_w(q_5, q_6, q_7)$ also stand for the same rotation matrix with which the value of q_5, q_6, q_7 can be calculated.

So, for a specific destination matrix and an Elbow Angle, solution of $Q = \begin{pmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 \end{pmatrix}^T$ can be get:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \\ \alpha_E \end{pmatrix} \quad (18)$$

and its differential formation can be written as:

$$\begin{pmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \\ \dot{q_4} \\ \dot{q_5} \\ \dot{q_6} \\ \dot{q_7} \end{pmatrix} = J(q_1, q_2, q_3, q_4, q_5, q_6, q_7) \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{\alpha_E} \end{pmatrix} \quad (19)$$

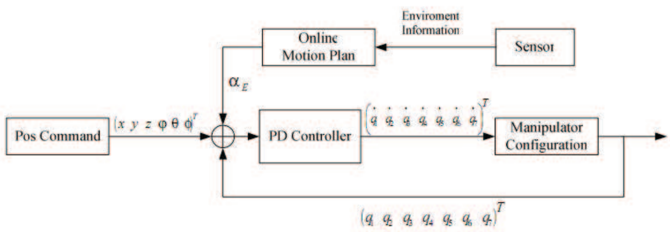

Fig. 6. Control Law of Online Motion Planning Algorithm

IV. Simulation Results

In the simulation, the end-effector of this manipulator is planned to follow a sine curve trajectory and the ELBOW angle is dynamically calculated based on the obstacle plane constrain. The sine curve of the end-effector is: $(x = 100 + \sin(t/8) \quad y = 150 + 2t \quad z = 1000)^T$ and the Rotation Matrix is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, and the constrain plane are combined with $10x+y+z+200 = 0$ and $10x-y+z+200 = 0$.

During this process this simulation, "elbow" of the manipulator is wish to always get attach with the constrain plane. Fig.7 shows the Elbow angle α_E which is decided by the constrain plane and the position and orientation of the end-effector discussed in III.D. And Fig.8, Fig.9, Fig.10 show the simulation result of the configuration of the manipulator, in which the top green line means the link4 of the manipulator; the red line and yellow line are link2 and link3; the last line means the link1. Besides, the blue sine curve is the trajectory of the end effector and the second blue curve is the path of "elbow" which is caused by Elbow angle α_E . These figures show that the end-effector successfully follow the sine curve and the "elbow" always get attach with the constrain plane.

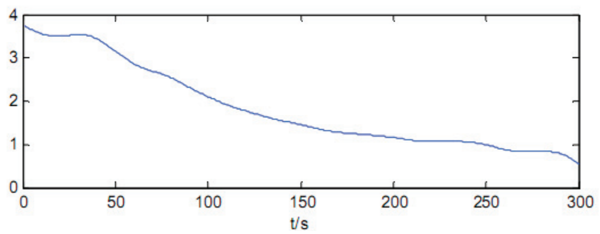

Fig. 7. Elbow Angle α_E

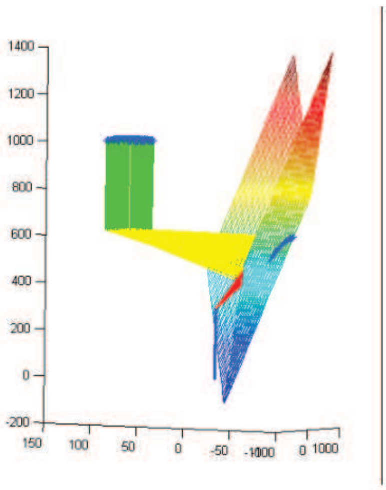

Fig. 8. Manipulator configuration a

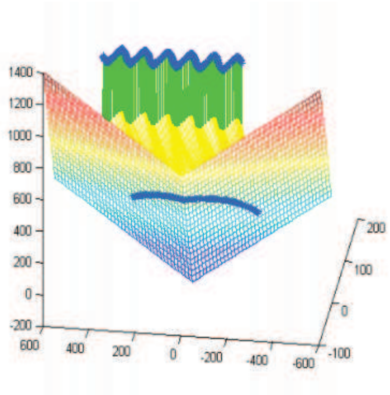

Fig. 9. Manipulator configuration b

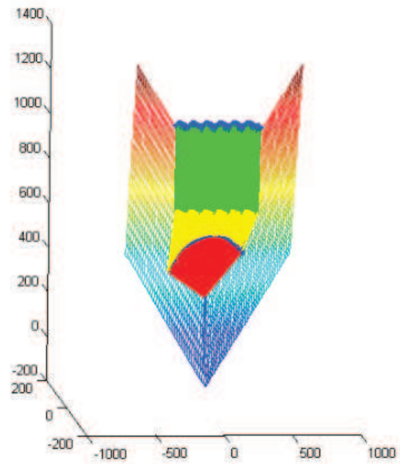

Fig. 10. Manipulator configuration c

V. Conclusion and Future Work

This paper proposes an online motion planning algorithm for this 7DOF manipulator by using a new inverse kinematic model. In the simulation, the end-effector is tracking a sine curve while the Elbow Angle is online decided by constrain planes build from environment information. The simulation results show that the inverse kinematic model is correct and motion of the redundant manipulator can autonomously planned online by environment information got by sensors. In this work, the plane constrain is a rough model of the environment information. In the future, we would like to move forward to find a better way to decide the Elbow Angle using sensor information. Besides, the Elbow Angle can also be decided by more criterions such like payload and energy saving.

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