Interval-Only Orientation Calculus with Asymmetric Granularity

by
Sree Harsha Ramanavarapu

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

April 2006

Interval-Only Orientation Calculus with Asymmetric Granularity

A Thesis Submitted
in Partial Fulfillment of the Requirements
for the Degree of
Master of Technology

by

Sree Harsha Ramanavarapu

to the

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY,KANPUR

April 2006

CERTIFICATE

It is certified that the work contained in the thesis entitled "*Interval-Only Orientation Calculus with Asymmetric Granularity*" by *Sree Harsha Ramanavarapu* has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

Dr. Amitabha Mukerjee
Department of Computer Science & Engineering,
Indian Institute of Technology,
Kanpur-208016.

Abstract

Qualitative orientation calculi attempt to capture only those orientational distinctions that are salient for reasoning about the physical world. Many calculi for orientation assume reflective symmetry - i.e. antipodal relations have equal ranges. Rules defined in natural language for human usage, for example navigation rules for vessels at sea, can be most flexibly expressed by an asymmetric calculus. However, most organisms have eyes in the "front" and hence a finer grained perception for front than for "back". In most proposed models one cannot encode such asymmetry. Furthermore, most qualitative orientation calculi propose ranges of orientations (intervals), and the boundaries between intervals are also labelled as "exact" or "ideal" relations. In practice (e.g. in implementations) however, such exact relations are zero probability events, and are associated with tolerances, thus introducing an extraneous interval at implementation time. Here we propose an interval-only calculus, *k-OPAA* (*k*-granularity Oriented Point Asymmetric Algebra), for relative orientation among oriented points. The granularity of the calculus can be arbitrary and asymmetric. *k-OPAA* gives the user freedom to choose how a 2*D* plane can be divided from an egocentric perspective. We also show the calculus is a relation algebra in the sense of Tarski and a Qualitative Calculus.

Acknowledgements

I would like to thank my guide, Prof. Amitabha Mukerjee for teaching me what research is all about. He often gave me the benefit of his experience by sharing useful tips and interesting anecdotes and in the process, he helped me understand the difference between developing a new concept and building a useful one. He also gave me an opportunity to work closely with research groups abroad, by sending me on an internship to the Spatial Cognition research group at the University of Bremen, Germany, which proved to be an invaluable learning experience.

A part of this work was carried out during a two-month internship at the University of Bremen, Germany. During that time, my supervisors, Dr. Christian Freksa and Mr. Frank Dylla gave me a lot of valuable guidance and feedback. In addition, they spent a considerable amount of time and effort to make our stay in Germany pleasant and comfortable. I thank them for making my internship an instructive and an extremely enjoyable experience. The internship was sponsored by DAAD (German Academic Exchange Service). Their support is duly acknowledged.

I would like to thank my friends Kranti Kumar and Arun Raghavan for their invaluable help and support throughout the course of this work. I would also like to thank my friends and classmates at IITK for all the fun we had together and for giving me two very good years.

Most importantly, I thank my family for their love and support, and I dedicate my efforts to them.

Contents

Chapter 1

Introduction

Qualitative reasoning seeks to make only those abstractions that are meaningful for reasoning about the physical world. Qualitative spatial representation assigns qualitative relations between spatial entities and their surroundings. The main motivation to use qualitative representation of space, is the human ability to perceive objects and space in qualitative rather than quantitative metrical terms. Spatial information is often represented in relative terms by humans, a good example of this being the vocabulary used to describe space in natural language (near, far, behind, across etc). Qualitative Spatial Reasoning (QSR) tries to reason with such qualitative relations.

Cohn and Hazarika in [CH01] give an overview of major qualitative spatial representation and reasoning techniques. They survey the main aspects of spatial representations and also consider methods for qualitative reasoning. Qualitative representations of space based on distance, orientation, shape etc., are discussed. Other areas for interest in QSR are robot navigation, computer vision, engineering design etc.

Spatial relations such as "front" or "near", which represent different categories of relations, may involve different precisions; thus "abuts" or "front" is nearly exact, while "overlaps" or "front-left" indicates a range or interval. In many formal algebras dealing with space, exact relations are often distinguished by constraints in a tighter dimension than that of the embedding space; e.g. for orientations, "front" may be an exact direction, while "front-left" permits a range of values [Fra91] [RM04] [MDF05] [MR08]. However, both cognitively[Lan03], [VSF+97] and algorithmmically [Gap94], it is clear that exact relations are almost always associated with some tolerances. Also, reasoning at different scales need to introduce intervals where there were only points. Cognitive experiments ([KDK+04], and also Section.1.2 below) also indicate that an adequate model to represent direction needs to have intervals, and not exact orientations, for representing directions like front, back, left and right.

However, since the exact relations in spatial calculi do not deal with these tolerances, substantial differences arise between theory and implementation. For example, a calculus may specify "front", as an exact direction, but an implementation may treat it as an angle between $\pm\epsilon$ degrees.

These are theoretical models and all the ramifications, in terms of accuracy, computational complexity, expressiveness etc., are all defeated in implementation. As an alternative we prove here a theoretical model, where every direction, whether front or front-left is associated with intervals.

Most spatial calculi dealing with orientations assume reflective symmetry, (relation (θ) exists then so does the relation ($\pi+\theta$)). Hence, for every discretization there is a converse. While such a converse relation is necessary for ensuring a relational algebra in the sense of Tarski, it has been shown that this is not the case for binary point relations, where a relation encodes both relative position/orientation of A with respect to B and for B with respect to A; in this situation the converse of a relation is just a swapping of the two parts of a relation [MDF05].

On the other hand, maintaining asymmetry is important for biological organisms, e.g. humans have finer grained perception in the front and little or no perception of what is behind them. Similarly, artificial agents such as wheeled robots and humanoids, may pay closer attention to objects in front than the back; so it would have a finer angular tolerance for relations towards the front than at the back, resulting in an asymmetric calculus. In the Section.1.2 we shall also report an experiment that evaluates the magnitude of disparity.

Here we propose a new qualitative spatial calculus for relative orientation, *k-OPAA* (*k*-granularity Oriented Point Asymmetric Algebra) which is an interval-only calculus with an arbitrary and asymmetric granularity. For any two objects A, B abstracted as oriented points, with an intrinsic reference direction (measured from an absolute framework), *k*-

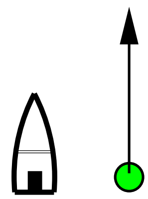
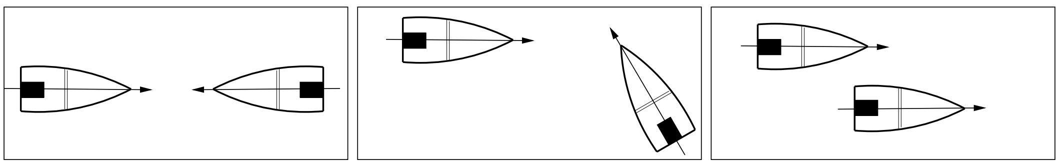


Figure 1.1: A boat abstracted to an *Oriented Point*



(a) Head-On (b) Crossing (c) Overtaking

Figure 1.2: Navigation Configurations: The distinction between Head-On and Crossing is crucial for many navigation rules.

OPAA encodes the orientation of A with respect to B and the orientation of B with respect to A in the same relation.

1.1 Reasoning with Orientation

Many spatial and orientation calculi simplify objects and locations by representing them as 2D points. We try to represent objects as points with orientation[MDF05]. Oriented points are related to concept of dipoles (objects as line segments) [MRW00].

Oriented points reduce objects with an intrinsic front to zero-length line segments with direction. In this conceptualization the size of the objects has no importance, since they are abstracted to point-objects with a direction. An object is represented by an *oriented point*, a 2D point and its direction in a 2D plane. Fig.1.1 abstracts a boat to an *oriented point* with 2D coordinates and a direction.

Objects can be abstractly represented by a point with an intrinsic direction (front) or an oriented point with a direction. An oriented point S, in a 2D plane P, is represented by global coordinates $p_S = (x_S, y_S)$, with $x_S, y_S \in \mathbb{R}$ and reference direction θ_S (also measured w.r.t a global framework), $S = (p_S, \theta_S)$.

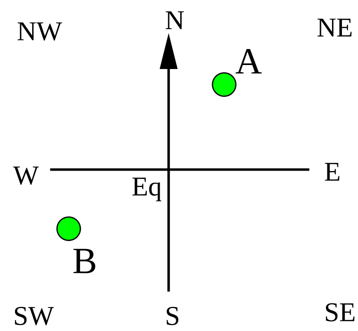


Figure 1.3: Cardinal Direction Calculus: *A* is to the North-East of *B* [Fra91]

Traffic rules for vessels at sea are specified by the *International Regulations for Preventing Collisions at Sea*, COLREGS (Collision Regulations) [Mal06] or by US National conventions [Gua99]. Navigation rules are described using configurations of vessels. Fig.1.1 describes three configurations which are distinguished in the rules, head-On (Fig.1.2a), overtaking (Fig.1.2c), crossing (Fig.1.2b). Expected behavior in the head-on case is different from that in the crossing case. These angular distinctions are therefore of great importance. Actions are specified for a vessel in each of these configurations, depending on its vessel type, direction of wind etc. Vessels at sea are abstracted to oriented points to reason about the orientations and relative positions of other vessels and maintain a rule-compliant behavior. In the following sections, we present another calculus for reasoning about such vessels, and introduce our model *k-OPAA*.

Existing calculi for spatial reasoning vary in their representation of exact relations or of symmetry [RS88] [Lev03]. Some calculi like *OPRA* ([MDF05]) and Cardinal Direction Calculus ([Fra91]) discretize orientation in terms of equal angles - but this is rarely how the world is broken up by a cognitive agent or by a task situation. The Cardinal Direction Calculus by Frank ([Fra91]) tries to represent the relative position of points in terms of global directions. Individual objects are considered direction-less entities. (See Fig.1.3)

Objects can also be represented as oriented line segment with two points (dipole). Each dipole is associated with a start and an end point, used to represent objects with an intrinsic front. The dipole calculus encodes relative orientation of two dipoles based on their start and end points.(See Fig.1.4). [DM05] used the dipole calculus to model agent behavior in dynamic environments. [MRW00]

Renz and Mitra in [RM04], propose the $STAR$ calculus, which permits an arbitrary

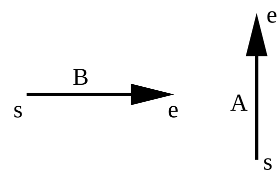
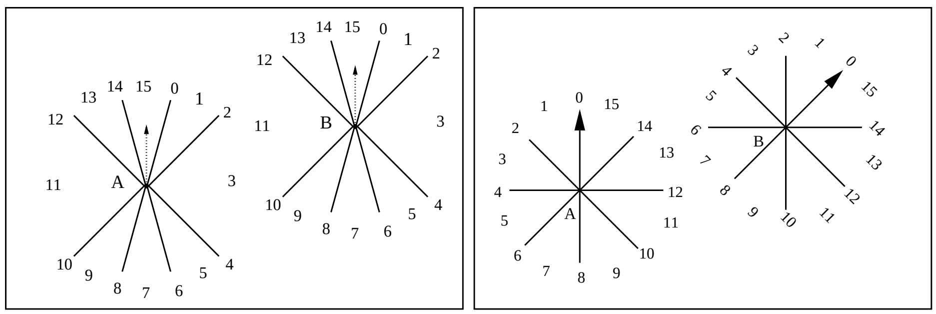


Figure 1.4: This relation is written as *A llrl B* in dipole calculus.



(a) $STAR_4[15, 45, 135, 165]$ (b) $OPRA_4$

Figure 1.5: Binary relation algebras: (a)All objects have the same(global) reference direction: A 3 B; B 11 A (b)Each object has its own intrinsic orientation(shown by arrow) $A_4\angle^{7}_{13}B$

discretization of orientations in terms of m intervals zones, bounded by m exact orientations (See Fig.1.5a). $STAR$ retains the reflective symmetry and all points are assumed to have the same global reference direction. Fig.1.5a shows two points A and B with a global reference direction and use $STAR_4[15, 45, 135, 165]$. The relation between A and B is 3 and relation between B and A is 11. B falls in region 3 of A and A falls in region 11 of B.

Unary orientation relations as in $STAR$ [RM04] and Cardinal Directions [Fra91] fail to handle egocentric relations, since the orientations are fixed along an an absolute frame of spatial reference, and are not relative [Lev03]. The $OPRA$ calculus proposed by Moratz et al. in [MDF05], handles this by maintaining binary relations, i.e. orientations of A w.r.t. B and B. w.r.t. A are both maintained (See Fig.1.5b). $OPRA$ divides the 360° circle divided into $2m$ uniform angle intervals, bounded $2m$ by exact orientations. An $OPRA$ relation between two oriented points A and B encodes the relative position of B with respect to A and relative position of A with respect to B. There are $(4m + 1) * 4m$ base relations including the identity relation in an $OPRA$ calculus with granularity m.

$OPRA$ and $STAR$ have intervals and exact orientations (lines) to represent relations; however, exact relations are unstable and donot to occur in practice.

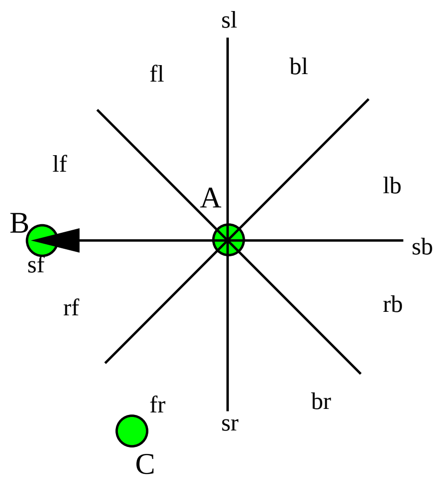


Figure 1.6: $TPCC$: $A\ B\ fr\ C$ (relation of between A and C in the direction of B is fr(front right)). Note that the boundary between lf and fl is not a relation, though that between lf and rf is.

An interesting dichotomy is observed in the *Ternary Point Configuration Calculus*($TPCC$) calculus proposed by Moratz and Ragni in [MR08] (Fig.1.6). The calculus considers orientation discretization with eight regions; some of the boundaries (e.g. front, left) as a labeled tessellations, whereas other boundaries like the lines at 45° are not; ie. one of the neighbouring relations is a half-open interval incorporating this line. In Fig.1.6 point *A* is the origin, *B* is relatum and *C* is the referent.

OPRA ideally replicates the ego-centric approach required when dealing with an application like robots in traffic. While this may seem like the perfect model at first sight, it still falls short of representing the asymmetry of human and humanoid vision [KDK+04]. Dividing the 360° into equal regions does not replicate this asymmetry. Unlike *OPRA*, *k-OPAA* gives the user complete freedom regarding the granularity of the calculus.

1.2 Asymmetry in human cognition

[VSF+97] considers the effects of direction of the axis with respect to its intrinsic front to explore how this affects decisions such as Front, Behind, Front-Left etc. Here subjects are tested with two experiments. In the first one, they are given a set of prepositions to choose from, to describe the relative position of a certain object with respect to a reference object.

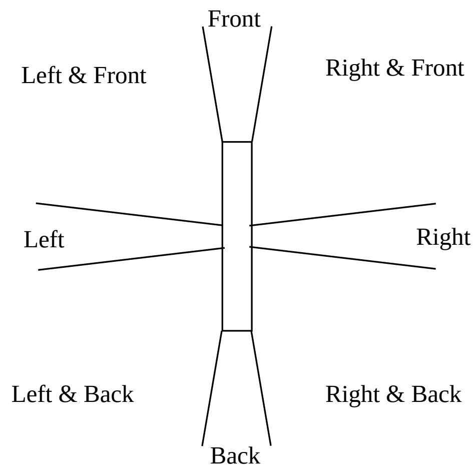


Figure 1.7: Prepositions chosen w.r.t reference object in [VSF+97]

In the second experiment, the subjects are allowed to freely describe the relative position of the object with respect to the referent. In both the experiments it was found that, subjects preferred to use two words like "front-left", "back-right" etc. Single word prepositions like "front", "right" etc. were used only in cases where the objects were considered to be perfectly aligned (See Fig.1.7). This shows that subjects were using axis imposed by the reference object.

[MGNE00] investigates the how size of the frame affects these direction terms and identify a "sweet spot". It is argued that direction terms like "in-front" do not discretize space into fixed quadrants based on the reference object's location. Direction terms are to be viewed as continuous fields within which points have differing levels of membership in a fuzzy class. To define the continuous fields, a single nonlinear function is used. Computer vision models are developed which take into account the effects of size, shape, motion, etc. which encode the ambiguity of conceptual descriptions.

Klippel et al. [KDK+04] investigate the concept of direction at intersections in street networks by asking users to choose various icons which symbolize different possibilities to make a turn. The results indicate that the 360 degrees being divided such that, Left and right are symmetric, front and back asymmetric. Verbal route directions and schematization of maps result in Fig.1.8.

While there is clear evidence of front-back asymmetry in the literature, in order to build a calculus, we need magnitudes. To get a measure of the asymmetry in human

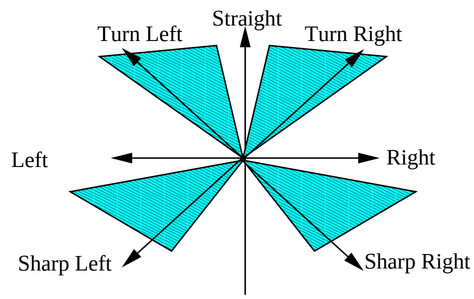


Figure 1.8: [KDK+04] shows that while left and right are symmetric, front and back are asymmetric.

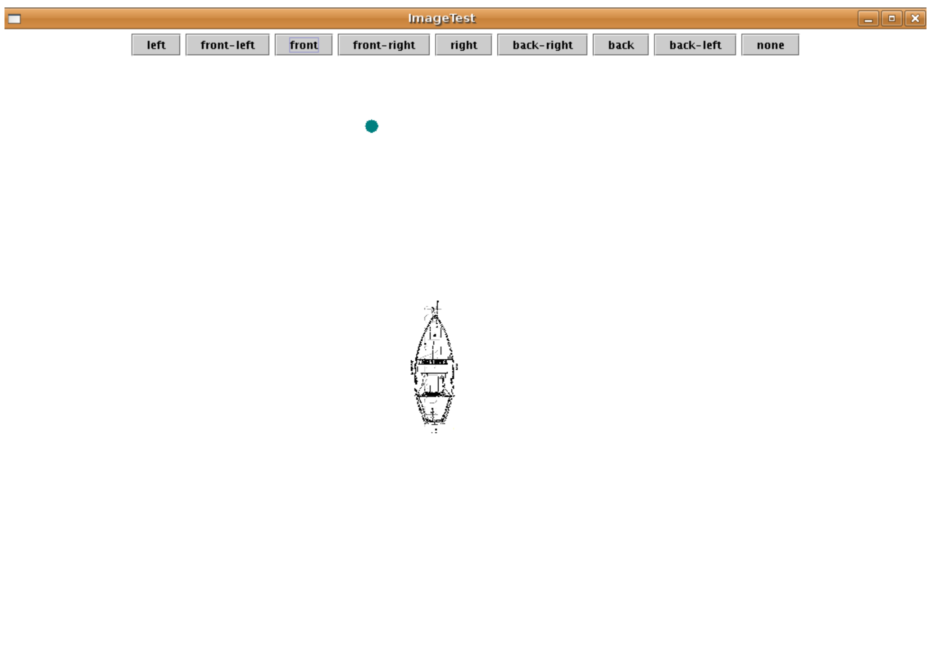


Figure 1.9: Subjects are asked imagine themselves piloting the boat. They need to choose the preferred direction term used to describe the position of the disk w.r.t themselves in the boat.

orientation cognition, we conduct a simple experiment to understand how front and back are perceived in the egocentric perspective. The subject is asked to classify a set of points with respect to a small boat, shown at the center of the screen. (See Fig.1.9) Small discs are shown at various orientations, a certain distance from the boat. The following instructions are read out to the user:

> There is a boat in the middle. You will be seeing a disk appear near the boat. If you are piloting the boat, would you consider the disk to be to the FRONT,

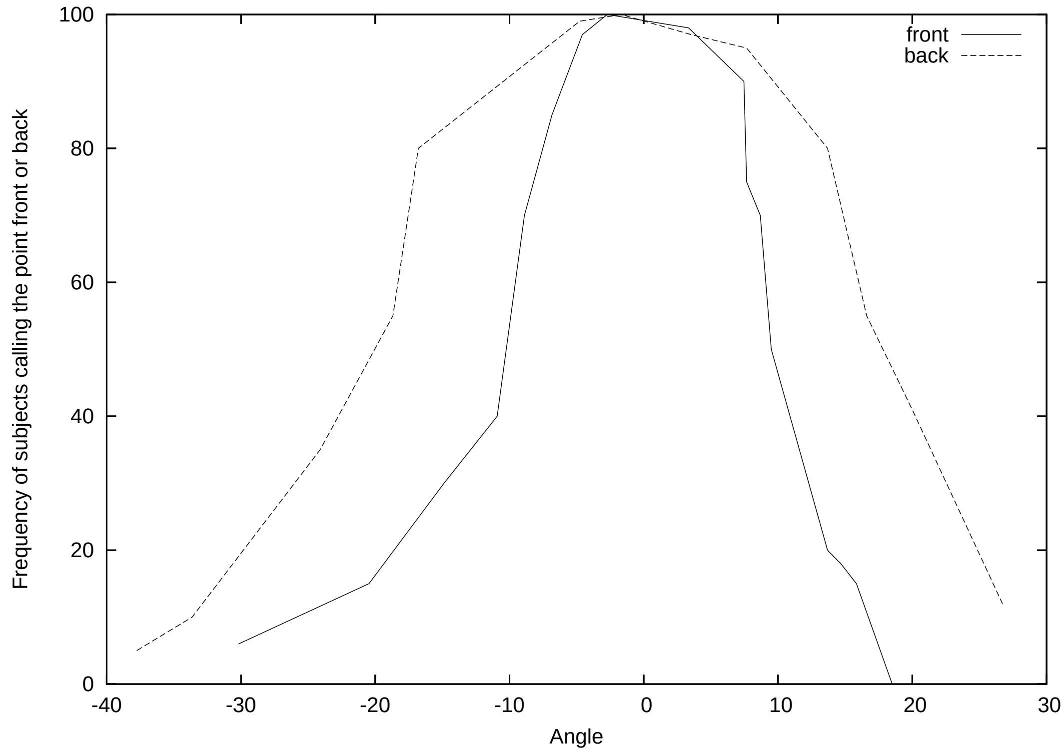


Figure 1.10: Values of Front(solid) are tighter than Back(dotted)

> LEFT etc. of your boat? Or is it in one of the in-between orientations like Front-left, Back-right etc?

The direction could be specified in terms of eight options (buttons): front, right, back, left, and front-right, front-left, back-right, back-left. The subject was given a trial run to get familiar with the setting, and then asked to classify a pre-programmed set of 30 disks.

Twenty subjects reported their perceptions. They reported the results verbally, while the experimenter clicked the menu to avoid false presses by some early subjects. The results (Fig.1.10) indicate a somewhat tighter distribution for the points in the Front direction (solid line curve, around θ=0), than in the Back direction (dashed line curve). While the sample size is somewhat small to make very strong conclusions, it does appear that there is a larger spread for the Back region, lending credence to the claim for asymmetry.

For a given application it may be appropriate to choose a frequency level (80%) and the angle ranges are front 17° and back 33°. Mean of front is 48.2 and the standard deviation 36.56 and the Mean of front is 58.15 and the standard deviation 33.14.

1.3 Thesis Organization

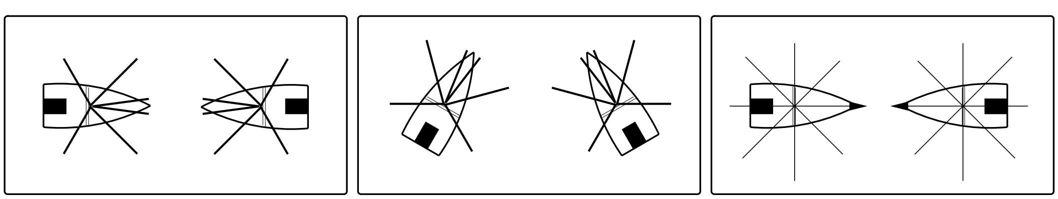
Chapter 2 gives the formal definition of the calculus *k-OPAA*. We also define the standard operators to be applied on a spatial calculus. Composition operator is explained in detail and an algorithm gives a comprehensive way to calculate the same. *k-OPAA* is shown to be a relation algebra in the sense of Tarski and is proved to be a qualitative spatial calculi. Chapter 3 gives the application of *k-OPAA* to the navigation simulator “Sailaway”. Transition sequences, which chart the relation trajectories of a vessel based on their actions are constructed for *k-OPAA*. Rule compliant behavior for vessels at sea is explained and the transition graphs help us in deciding whether a certain vessel is rule compliant or not. The architecture of the simulator is explained and the reactive behavior is also encoded.

Chapter 2

k-OPAA Calculus

The *k-OPAA* (*k*-granularity Oriented Point Asymmetric Algebra) calculus deals with relative positions of oriented points by representing the relations of both points in the intrinsic frames of each other. A vessel is abstracted to an *oriented point*, $2D$ point with its front being the reference direction θ with respect to a global framework. A typical configuration like *head-on* (Fig.2.1a) the 6-$OPAA[5,45,120,240,315,355]$ relation is $\angle_0^0$. The $OPRA_4$ relation ${}_4\angle_0^0$ (see Fig.2.1c) never arises, since 0 is an exact relation, this is a zero-probability event. In contrast, using 6-$OPAA[5,45,120,240,315,355]$ gives us the direction front associated with a tolerance of 10°.

[MDF05] also gives a simple path based example, where the robot uses a qualitative orientation calculi to move from one place to another. The robot reasons about its surroundings using standard constraint propagation techniques to avoid collisions and incrementally moves towards its destination. A similar example, can be used for reasoning with *k-OPAA*.



(a) 6-$OPAA$: $\angle_0^0$ (b) 6-$OPAA$: $\angle_4^2$ (c) $OPRA_4$: ${}_4\angle_0^0$

Figure 2.1: Vessels are head-on

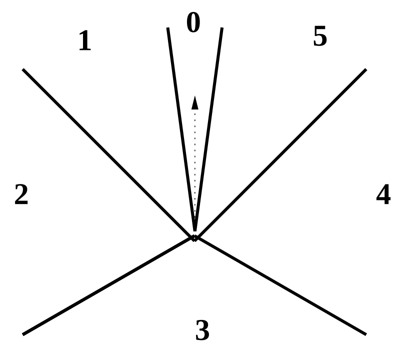


Figure 2.2: 6-*OPAA*$[5, 45, 120, 240, 315, 355]$. The intervals are labeled starting with zero, so interval 3 spans the sector [120,240).

2.1 *k-OPAA* Calculus: Definition

Given a two-dimensional plane P, for each oriented point $S \in P$, $S = (p_S, \theta_S) = ((x_S, y_S), \theta_S)$, *with the calculus m-OPAA*$[\delta_0, \ldots, \delta_{m-1}]$ *where* $0 \leq \delta_0 < \delta_1 < \delta_2 \ldots < \delta_{m-1} \leq 360$*, specifies m half-lines (rays) which intersect at* S*. Each of the m rays make an angle* δ_j *(*$0 \leq j < m$*) with its reference direction,* θ_S*.*

For any point $S \in P$*, these m rays partition P into m disjoint angular zones with respect to its reference direction. Intervals are numbered in the counter-clockwise direction,* $0, 1, \ldots, m-1$*. The interval from* $[\delta_{m-1}, \delta_0)$ *is numbered* 0*, interval from* $[\delta_0, \delta_1)$ *is* $1, \ldots$ *and interval from* $[\delta_{m-2}, \delta_{m-1})$ *is* $m-1$*. A* $2D$ *interval numbered i, spans the sector* $[\delta_{i-1}, \delta_i)$*.*

Fig.2.2 represents the calculus 6-*OPAA*$[5, 45, 120, 240, 315, 355]$ (the dotted line represents the reference direction). Interval 0 spans the sector $[\delta_{m-1}, \delta_0)$, i.e. any point in 0 makes an angle A_0 with the reference direction such that $355^\circ \leq A_0 < 5^\circ$. Similarly, interval 1 spans the sector $[\delta_0, \delta_1)$, i.e. any point in 1 makes an angle A_1 with the reference direction such that $5^\circ \leq A_1 < 45^\circ$. Interval 5 spans the sector $[\delta_4, \delta_5)$, i.e. any point in 5 makes an angle A_5 with the reference direction such that $315^\circ \leq A_5 < 355^\circ$.

The set of all base relations, K, between two points $A = (p_A, \theta_A)$ and $B = (p_B, \theta_B)$ each with a calculus *m-OPAA*$[\delta_0, \delta_1, \ldots, \delta_{m-1}]$ are defined as,

- the identity relation $id \equiv \{p_A = p_B, \theta_A = \theta_B\}$.
- if A and B have equal cartesian coordinates $p_A = p_B$, but different reference directions $\theta_A \neq \theta_B$, then $A \prec_i^j B$ ($0 \leq i, j \leq m-1$) (see Equation 2.2).

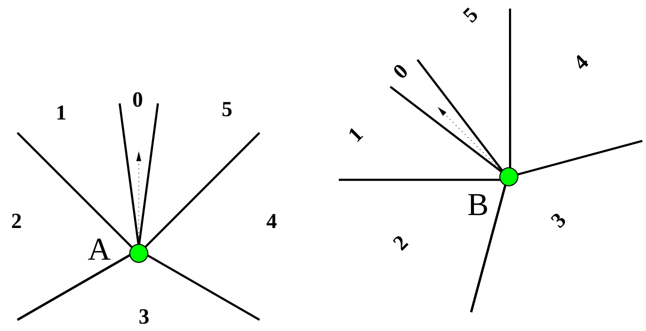


Figure 2.3: $rel_{AB} = A\angle_4^2 B$

- if A and B have different cartesian coordinates $p_A \neq p_B$, relation is $A\angle_i^j B$ (Fig.2.3). B lies in sector i of A and A lies in sector j of B ($0 \leq i, j \leq m-1$) (see Equation 2.1).

The set of all k-*OPAA* relations, $\mathbb{K}$ is the power set of K.

In Fig.2.3 the oriented points A and B are associated with 6-*OPAA*$[5, 45, 120, 240, 315, 355]$. rel_{AB} gives the relations between A and B as $A\angle_4^2 B$, i.e. the relative position of B with respect to A falls in the interval 4 and the relative position of A with respect to B falls in the interval 2.

$\ominus$ represents a cyclic subtraction on $\mathbb{R}_{360}$. ϕ_{AB} represents the direction of $\overrightarrow{AB}$ with respect to the reference direction of A, θ_A and ϕ_{BA} represents the direction of $\overrightarrow{BA}$ with respect to the reference direction of B, θ_B.

Formally, two points A, B have different cartesian coordinates ($p_A \neq p_B$) and are represented by the calculi m-*OPAA*$[\delta_0, \ldots, \delta_{m-1}]$, then $rel_{AB} = A\angle_i^j B$ ($0 \leq i, j \leq m-1$),

$$(\delta_{i-1} \leq \phi_{AB} \ominus \theta_A < \delta_i) \wedge (\delta_{j-1} \leq \phi_{BA} \ominus \theta_B < \delta_j) \tag{2.1}$$

m-*OPAA*$[\delta_0, \ldots, \delta_{m-1}]$ has m^2 base relations if $p_A \neq p_B$.

If A and B have same cartesian coordinates ($p_A = p_B$) but different reference directions ($\theta_A \neq \theta_B$), $rel_{AB} = A \prec_i^j B$ ($0 \leq i, j \leq m-1$),

$$(\delta_{i-1} \leq \theta_B \ominus \theta_A < \delta_i) \wedge (\delta_{j-1} \leq \theta_A \ominus \theta_B < \delta_j) \tag{2.2}$$

If $\delta_i \ominus \delta_{i-1} < \delta_j \ominus \delta_{j-1}$, the value of j can be pre-decided by $\delta_{j-1} = \delta_{i-1} + \theta_A \ominus \theta_B$ and

$\delta_j = \delta_i + \theta_A \ominus \theta_B$. Hence, m-$OPAA[\delta_0, \ldots, \delta_{m-1}]$ has m base relations if $p_A = p_B$ and $(\theta_A \neq \theta_B)$.

There is only one identity relation, id ($p_A = p_B$ and $\theta_A = \theta_B$). Hence, m-$OPAA[\delta_0, \ldots, \delta_{m-1}]$ has $m^2 + m + 1$ base relations.

Satisifying the following three conditions makes it a Qualitative Calculus in the sense of [LR04]:

- The set of all *base relations* should be Jointly Exhaustive and Pairwise Disjoint (JEPD). A set of base relations, K, is JEPD if, $\forall x, y$ (x, y are oriented points), there is a single relation $\{xRy \mid R \in K\}$.
- The calculus should be closed under the operators: Union ($\cup$), Intersection ($\cap$), Converse ($\smile$), Complement ($'$) and Composition ($\circ$). The set of relations should also contain an Empty relation ($\{\}$), an Identity relation (id) and a Universal relation ($\bigsqcup$). See Section.2.2.
- The calculus should be a relation algebra in the sense of Tarski. Constraint based reasoning techniques used for spatial relations as introduced by Ladkin and Maddux in [LM94], are applicable to relation algebras. See Section.2.4

2.2 Operators

k-OPAA calculus is closed under the operators ($\cup$, $\cap$, $\smile$, $'$, $\circ$).

- Empty relation - $\{\}$ - is the null set.
- Identity relation - $id \equiv \{p_A = p_B, \theta_A = \theta_B\}$
- Universal relation - $\bigsqcup$ - is the union of all base relations.

The operators Union ($\cup$), Intersection ($\cap$), Complement ($'$) can be computed using regular set-theoretic definitions of the relations.

1. Converse Operator ($\smile$):

- If $rel_{AB} = id$ then $rel_{BA} = rel_{AB}^{\smile} = id$.
- If $rel_{AB} =\prec_i^j$ then $rel_{BA} = rel_{AB}^{\smile} =\prec_j^i$, $((p_A = p_B), \theta_A \neq \theta_B)$.
- If $rel_{AB} = \angle_i^j$, then $rel_{BA} = rel_{AB}^{\smile} = \angle_j^i$, $(p_A \neq p_B)$.

2. Composition Operator ($\circ$): The composition operator output is defined in terms of the variables s , t which will be obtained in the next section.

- $id \circ rel_{AB} = rel_{AB} \circ id = rel_{AB}$
- If $rel_{AB} = \angle_i^j, rel_{BC} = \angle_k^l$, then $rel_{AC} = \angle_t^s$.(See Section 2.3.1 (Case i))
- If $rel_{AB} = \angle_i^j, rel_{BC} =\prec_k^l$, then $rel_{AC} = \angle_t^s$.(See Section 2.3.2 (Case ii))
- If $rel_{AB} =\prec_i^j, rel_{BC} =\prec_k^l$, then $rel_{AC} =\prec_t^s$. (See Section 2.3.3 (Case iii))

2.3 Composition Operator

Composition operator is explained using the simple geometric semantics of the *k-OPAA* relations. Composition is defined only for the base relations. The methods that we have used are very close to the composition operator in *OPRA*[FLWD07]. Consider three points $A = (p_A, \theta_A)$, $B = (p_B, \theta_B)$, $C = (p_C, \theta_C)$.

2.3.1 Case i: ($rel_{AB} = \angle_i^j$, $rel_{BC} = \angle_k^l$, $rel_{AC} = \angle_t^s$)

When all three points have different cartesian coordinates,
$p_A \neq p_B \neq p_C$, See Fig.2.4. We need to determine $rel_{AC} = \angle_t^s$, given rel_{AB} and rel_{BC}.

Fig.2.5 introduces the triangle constraint. A_B represents the direction of A oriented towards B and A_C representing the the direction of A oriented towards C. α is the angle between A_B and A_C. Similarly, β is the angle between B_A and B_C and γ is the angle between C_A and C_B. Since A, B, C are distinct points, they can form a triangle with α, β and γ as its angles. Hence $\alpha + \beta + \gamma = 180°$. [FLWD07]

Given rel_{AB} and rel_{BC}, we need to find rel_{AC}, i.e., we need to find the values of t and s when i, j, k and l are given. We know the relative position of $\overrightarrow{B}$ with respect to $\overrightarrow{A}$ from i

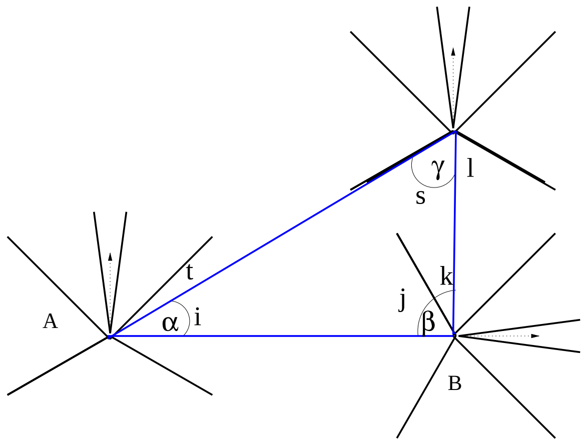


Figure 2.4: Triangle ABC- $\alpha + \beta + \gamma = 180$ (follows from [FLWD07])

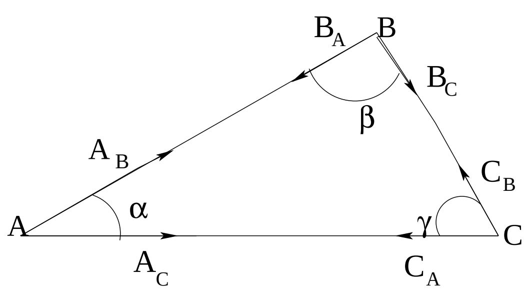


Figure 2.5: Triangle ABC (follows from [FLWD07])

and the relative position of $\overrightarrow{A}$ with respect to $\overrightarrow{B}$ from j. This can be represented by,

$$\alpha = \delta_t \ominus \delta_i, \quad \beta = \delta_j \ominus \delta_k, \quad \gamma = \delta_l \ominus \delta_s$$

Here the values of i and t denote the intervals of $\overrightarrow{A}$, j and k denote sectors of $\overrightarrow{B}$, l and s denote sectors of $\overrightarrow{C}$. To handle this, we represent α is bounded by $[\alpha_1, \alpha_2)$ in the positive orientation and by $[\alpha_3, \alpha_4)$ in the negative orientation. Similarly, β and γ are bounded by $[\beta_1, \beta_2)$, $[\gamma_1,\gamma_2)$, in the positive orientation and by $[\beta_3, \beta_4)$, $[\gamma_3,\gamma_4)$, in the negative orientation respectively.

$$\begin{aligned}
\alpha_1 &= \delta_{t-1} \ominus \delta_i, \quad \alpha_2 = \delta_t \ominus \delta_{i-1},\\
\alpha_3 &= \delta_{i-1} \ominus \delta_t, \quad \alpha_4 = \delta_i \ominus \delta_{t-1}\\
\beta_1 &= \delta_{j-1} \ominus \delta_k, \quad \beta_2 = \delta_j \ominus \delta_{k-1},\\
\beta_3 &= \delta_{k-1} \ominus \delta_j, \quad \beta_4 = \delta_k \ominus \delta_{j-1}\\
\gamma_1 &= \delta_{l-1} \ominus \delta_s, \quad \gamma_2 = \delta_l \ominus \delta_{s-1},\\
\gamma_3 &= \delta_{s-1} \ominus \delta_l, \quad \gamma_4 = \delta_s \ominus \delta_{l-1}
\end{aligned}$$

In the case of positive orientation, $\alpha \in [\alpha_1, \alpha_2), \beta \in [\beta_1,\beta_2), \gamma \in [\gamma_1, \gamma_2)$. Hence

$$\begin{aligned}
&\alpha + \beta + \gamma = \pi\\
\Rightarrow \quad &\pi \in [\alpha_1 + \beta_1 + \gamma_1, \alpha_2 + \beta_2 + \gamma_2)
\end{aligned}$$

In the case of negative orientation, $\alpha \in [\alpha_3, \alpha_4), \beta \in [\beta_3,\beta_4), \gamma \in [\gamma_3, \gamma_4)$. Hence

$$\pi \in [\alpha_3 + \beta_3 + \gamma_3, \alpha_4 + \beta_4 + \gamma_4)$$

2.3.2 Case ii: $(rel_{AB} =\prec_i^j, rel_{BC} = \angle_k^l, rel_{AC} = \angle_t^s)$

When two points have the same cartesian coordinates, and one is different, $p_A = p_B \neq p_C$, See Fig.2.6. We need to determine $rel_{AC} = \angle_t^s$, given rel_{AB} and rel_{BC}.

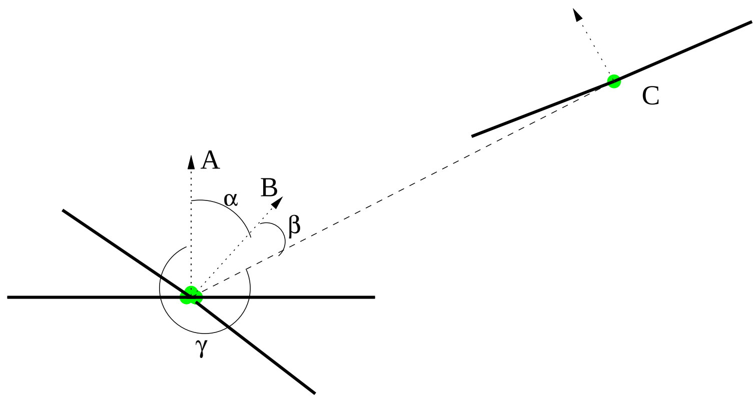


Figure 2.6: Composition with $p_A = p_B \neq p_C$ (follows from [FLWD07])

$$\alpha = \delta_i, \beta = \delta_j \ominus \delta_k, \gamma = \delta_l \ominus \delta_s$$

$$\begin{aligned} \alpha_1 &= \delta_{i-1}, & \alpha_2 &= \delta_i, \\ \alpha_3 &= \delta_{i-1}, & \alpha_4 &= \delta_i \\ \beta_1 &= \delta_{j-1} \ominus \delta_k, & \beta_2 &= \delta_j \ominus \delta_{k-1}, \\ \beta_3 &= \delta_{k-1} \ominus \delta_j, & \beta_4 &= \delta_k \ominus \delta_{j-1} \\ \gamma_1 &= \delta_{l-1} \ominus \delta_s, & \gamma_2 &= \delta_l \ominus \delta_{s-1}, \\ \gamma_3 &= \delta_{s-1} \ominus \delta_l, & \gamma_4 &= \delta_s \ominus \delta_{l-1} \end{aligned}$$

In the case of positive orientation, $\alpha + \beta + \gamma = 2\pi$,

$$2\pi \in [\alpha_1 + \beta_1 + \gamma_1, \alpha_2 + \beta_2 + \gamma_2)$$

In the case of negative orientation $2\pi \in [\alpha_3 + \beta_3 + \gamma_3, \alpha_4 + \beta_4 + \gamma_4)$.

2.3.3 Case iii: ($rel_{AB} = \prec_i^j$, $rel_{BC} = \prec_k^l$,$rel_{AC} = \prec_t^s$)

When all three points have the same cartesian coordinates,
$p_A = p_B = p_C$, See Fig.2.7. We need to determine $rel_{AC} = \prec_t^s$, given rel_{AB} and rel_{BC}.

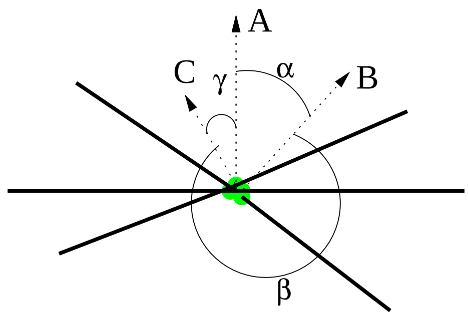


Figure 2.7: Composition with $p_A = p_B = p_C$ (follows from [FLWD07])

$$\alpha_1 = \delta_{i-1}, \quad \alpha_2 = \delta_i,$$
$$\beta_1 = \delta_{k-1}, \quad \beta_1 = \delta_k,$$
$$\gamma_1 = \delta_{l-1}, \quad \gamma_1 = \delta_l,$$

$$\alpha + \beta + \gamma = 2\pi$$
$$\Rightarrow \quad 2\pi \in [\alpha_1 + \beta_1 + \gamma_1, \alpha_2 + \beta_2 + \gamma_2)$$

2.3.4 Algorithm for Composition Operator

The algorithm tests the existence of the triangle (ABC) with all possible values of t and s ($rel_{AC} = \angle_t^s$) for the given $\{i, j, k, l\}$. See Algorithm.1. It is an $O(k^2)$ complexity algorithm, where k is the granularity of the corresponding k-$OPAA$.

2.4 Reasoning with k-$OPAA$

Let K be the set of all base relations of k-$OPAA$, and $\mathbb{K}$ is the power set of K.

Algorithm 1 Composition (rel_{AB}, rel_{BC})

```
if $rel_{AB} = id \vee rel_{BC} = id$ then                ▷ one $id$ relation
    $\angle_t^s = id \circ R = R \circ id = R$
else
    for $t = 0$ to k-1 do
        for $s = 0$ to k-1 do
            if $rel_{AB} = \prec_i^j \wedge rel_{BC} = \prec_k^l$ then        ▷ two $\prec$ relations
                See case iii
            else
                if $rel_{AB} = \prec_i^j \vee rel_{BC} = \prec_k^l$ then        ▷ one $\prec$ relation
                    See case ii
                else        ▷ case i - Only Positive orientation mentioned
                    $\alpha_1 = \delta_{t-1}$ - $\delta_i$ $\alpha_2 = \delta_t$ - $\delta_{i-1}$
                    $\beta_1 = \delta_{j-1}$ - $\delta_k$ $\beta_2 = \delta_j$ - $\delta_{k-1}$
                    $\gamma_1 = \delta_{l-1}$ - $\delta_s$ $\gamma_2 = \delta_l$ - $\delta_{s-1}$

                    if $\alpha_1 + \beta_1 + \gamma_1 \leq \pi < \alpha_2 + \beta_2 + \gamma_2$ then
                        $rel_{AC} = rel_{AC} \cup \angle_t^s$
                    end if
                end if
            end if
        end for
    end for
end if
```

2.4.1 *k-OPAA* as Relation Algebra

$r_i, r_j \in K$ are base relations. Compound relations, $R_1, R_2 \in \mathbb{K}$. For $r_i \in R_1, r_j \in R_2$,

$$(R_1 \circ R_2) = \bigcup_{i,j}(r_i \circ r_j) \tag{2.3}$$

$$\begin{aligned}(R_1 \cup R_2)^{\smile} = & \bigcup_{i,j}(r_i \cup r_j)^{\smile} \\ & = \bigcup_{i,j}(r_i^{\smile} \cup r_j^{\smile})\end{aligned} \tag{2.4}$$

The relation algebra $\mathbf{K} = (\mathbb{K}, \cup, \cap, \smile, ', \circ, \{\}, id, \bigsqcup)$.

To prove that *k-OPAA* ($\mathbf{K}$) is a relation algebra in the sense of Tarski, the following conditions must be satisfied [Mad06].

- **Rule 1: Commutative**:- $(R_1 \cup R_2) = (R_2 \cup R_1)$
  $(rel_{AB} \cup rel_{CD}) = (rel_{CD} \cup rel_{AB})$.
- **Rule 2: Associative**:- $(R_1 \cup R_2) \cup R_3 = R_1 \cup (R_2 \cup R_3)$
  $(rel_{AB} \cup rel_{CD}) \cup rel_{EF} = rel_{AB} \cup (rel_{CD} \cup rel_{EF})$
- **Rule 3: Huntington's Axiom**:- $(R_1' \cup R_2')' \cup (R_1' \cup R_2)' = R_1'$
  $(rel_{AB}' \cup rel_{CD}')' \cup (rel_{AB}' \cup rel_{CD})'$
  $= ((rel_{AB}')' \cap (rel_{CD}')') \cup ((rel_{AB}')' \cap rel_{CD}')$
  $= (rel_{AB} \cap rel_{CD}) \cup (rel_{AB} \cap rel_{CD}')$
  $= rel_{AB}$
- **Rule 4: Associativity**:- $(R_1 \circ R_2) \circ R_3 = R_1 \circ (R_2 \circ R_3)$
  $(rel_{AB} \circ rel_{BC}) \circ rel_{EF} = rel_{AB} \circ (rel_{BC} \circ rel_{EF})$
- **Rule 5: Distributivity**:- $(R_1 \cup R_2) \circ R_3 = R_1 \cup (R_2 \circ R_3)$
  $(rel_{AB} \cup rel_{BC}) \circ rel_{EF} = rel_{AB} \circ (rel_{BC} \circ rel_{EF})$ (see Equation.2.3)
- **Rule 6: Identity Law**:- $id \circ R = R \circ id = R$
  $id \circ rel_{AB} = rel_{AB} \circ id = rel_{AB}$

- **Rule 7:Involution**:- $(R^{\smile})^{\smile} = R$
  $rel_{AB} = id$, $((rel_{AB})^{\smile})^{\smile} = (id)^{\smile} = id$
  $rel_{AB} =\prec_i^j$, $((rel_{AB})^{\smile})^{\smile} = (\prec_j^i)^{\smile} =\prec_i^j$
  $rel_{AB} = \angle_i^j$, $((rel_{AB})^{\smile})^{\smile} = (\angle_j^i)^{\smile} = \angle_i^j$

- **Rule 8:Distributivity**:- $(R_1 \cup R_2)^{\smile} = R_1^{\smile} \cup R_2^{\smile}$
  $(rel_{AB} \cup rel_{CD})^{\smile} = rel_{AB}^{\smile} \cup rel_{CD}^{\smile}$ (see Equation.2.4)

- **Rule 9: Involute Distributivity**:- $(R_1 \circ R_2)^{\smile} = R_2^{\smile} \circ R_1^{\smile}$
  $(rel_{AB} \circ rel_{BC})^{\smile} = rel_{AC}^{\smile} = rel_{CA}$,
  $rel_{BC}^{\smile} \circ rel_{AB}^{\smile} = rel_{CB} \circ rel_{BA} = rel_{CA}$

- **Rule 10: Tarski/De Morgan Axiom**:- $R_1^{\smile} \circ (R_1 \circ R_2') \leq R_2'$
  Here $x \leq y \Leftrightarrow x \cup y = y$. Rules 1,2 and 3 are necessary for **K** to be a boolean algebra. **K** is also a boolean algebra with the monoid taken to be a conjunction, hence it is a residuated boolean algebra. Residuated boolean algebra is a relation algebra according to [JT93].

2.4.2 Constraint Satisfaction

Given a set of n oriented points $\{S_1, S_2, \ldots, S_n\}$, and a finite set of binary constraints Θ, $S_x rel_{xy} S_y$ such that $S_x, S_y \in S$ and $rel_{xy} \in \mathbb{B}$. The *Constraint Satisfaction Problem* (CSP), deals with finding atleast one instance of n oriented points, which satisfy all the constraints (Θ). The constraint network is a graph, with oriented points $(S_x, S_y, S_z \ldots)$, as nodes and the binary relations are the edges $(rel_{xy}, rel_{xz}, rel_{zy}, \ldots)$. The CSP for *k-OPAA* is similar to $OPSAT$, defined by [MDF05]. $OPSAT$ can be solved using the standard constraint network techniques with infinite domains mentioned by [LM94].

A necessary condition for proving the consistency of the CSP was proposed by Mackworth in [Mac77], the *path-consistency* method. A network is path-consistent if, for any three nodes S_x, S_y, S_z, there exists a relation on the edge (S_x, S_y) and is a subset of the composition of relations between (S_x, S_z) and (S_z, S_y). Given two nodes $S_x, S_y \in S$ with edge (relation) rel_{xy}, then we need to find a node $S_z \in S$ such that,

$$rel_{xy} \leftarrow rel_{xy} \cap (rel_{xz} \circ rel_{zy})$$

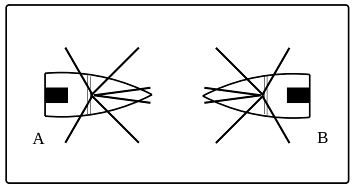


Figure 2.8: Two boats abstracted to oriented points and each associated with 6-$OPAA[5, 45, 120, 240, 315, 355]$.

where $rel_{xy}, rel_{xz}, rel_{zy} \in \mathbb{B}$. The path-consistency method in the context of spatial reasoning is well explained by Renz and Nebel in [RN99].

Allen proposed an *NP-Complete* solution for path-consistency, it can be determined in $O(n^3)$ time (n is the number of nodes in the constraint network). If the resulting CSP has the same relations as the original, it is path-consistent. If an empty relation occurs between two nodes, the CSP is inconsistent.[All83]

2.4.3 Conceptual Neighborhood

The notion of conceptual neighborhood has been introduced by Freksa [Fre91]. Two relations of a qualitative spatial calculus are conceptual neighbors if they can be continuously transformed into each-other without resulting in a third relation in between.

According to Equation.2.5, the conceptual neighbors of $\angle_0^0$ are $\{\angle_1^0, \angle_5^0, \angle_0^1, \angle_0^5, \angle_5^5, \angle_1^5, \angle_1^1, \angle_5^1\}$. [DFW+07] [DW07]

$$cn(\angle_i^j) = \{\angle_{i+1}^j, \angle_{i-1}^j, \angle_i^{j+1}, \angle_i^{j-1}, \angle_{i-1}^{j-1}, \angle_{i+1}^{j-1}, \angle_{i+1}^{j+1}, \angle_{i-1}^{j+1}\} \tag{2.5}$$

Assume that two boats A and B, with intrinsic fronts, are abstracted to oriented points. Each point is associated with 6-$OPAA[5, 45, 120, 240, 315, 355]$. As show in Fig.2.8, the two boats have the relation $A\angle_0^0 B$.

Let us try to understand how the rotating objects will affect the relation. When object A turns right, the relative position of B will fall in the angular zone numbered 1. Conversely, when object A turns left, the relative position of B will fall in the angular zone numbered 5 (cyclic subtraction).[DW07]

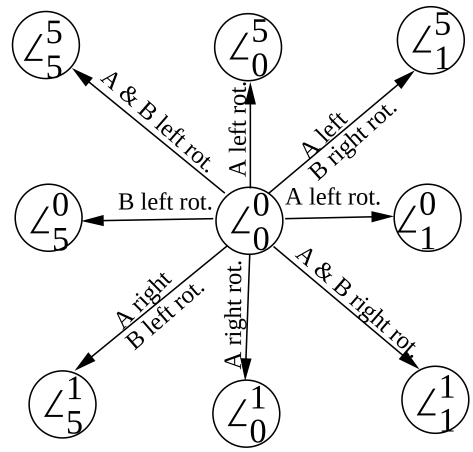


Figure 2.9: 6-*OPAA*$[5,45,120,240,315,355]$ - Possible neighborhood transitions $\angle_0^0$ when objects rotate ([DW07]).

Fig.2.9 illustrates the neighborhood transitions for the relation $\angle_0^0$ based on actions. We use transition graphs which represents the 6-*OPAA* relations as nodes and the actions as edges. Using this we can determine whether a certain set of actions lead to a certain relation.

Chapter 3

Qualitative Spatial Reasoning with *k-OPAA*

The *k-OPAA* calculus deals with relative positions of oriented points by representing the relations of both points in the intrinsic frames of each other. This treatment is similar to *OPRA* (Oriented Point Relation Algebra) . For comparison, we consider using *k-OPAA*, for in an application which uses *OPRA*, the formalization of navigation rules for vessels at sea[DFW+07]. Traffic rules for vessels at sea are specified by the International Regulations for Preventing Collisions at Sea, COLREGS (Collision Regulations) of the International Maritime Organization (IMO) [Mal06] [Gua99].

Navigation rules are encoded using *OPRA* to represent configurations (spatial constellation of vessels) by [DFW+07]. Given the orientation relation between two vessels in *OPRA* terms, a set of rules are defined, to map a situation to its corresponding configuration

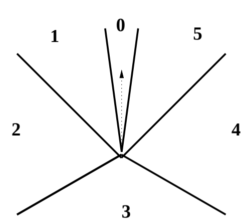


Figure 3.1: 6-*OPAA*$[5, 45, 120, 240, 315, 355]$

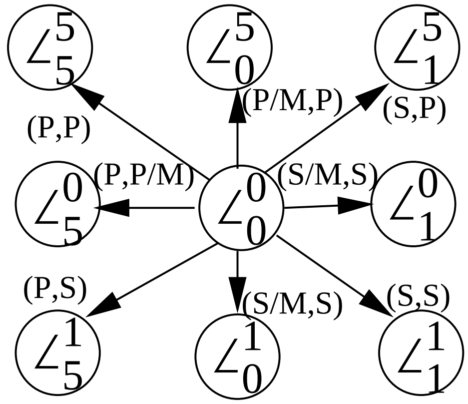


Figure 3.2: Neighborhood transitions based on actions from 6-*OPAA*$[5, 45, 120, 240, 315, 355]$ relation $\angle_0^0$

specified by the COLREGS. Rule compliant behavior of a vessel can be verified with the transitions of *OPRA* relations. The actions are, turn starboard (turn right, represented by S), turn port (turn left, represented by P), keep midships (stay the course, represented by M).

Actions are performed over a period of time. Temporal information can be integrated in qualitative spatial representation by using conceptual neighborhoods as described in [Fre91] [DW07]. The movement of an agent can be modeled as a sequence of neighboring spatial relations which hold for adjacent time intervals. Using this qualitative representation of trajectories, neighborhood-based spatial reasoning can be used as a simple, abstract model of agent navigation [DM05].

According to the Equation.2.5, the relation $\angle_0^0$ of has $\angle_0^1$, $\angle_1^1$, $\angle_1^0$, $\angle_0^5$, $\angle_5^5$, $\angle_5^0$, $\angle_5^1$ and $\angle_1^5$ as its neighbors (Assume we are using the calculus from Fig.3.1). A change in direction or position can lead to a different relation. This is represented graph in Fig.3.2, nodes contain the relation and the edges represent the actions.

When two vessels have a 6-*OPAA*$[5, 45, 120, 240, 315, 355]$ relation $\angle_0^0$, if both turn starboard (represented by (S, S)), they *may* reach the relation $\angle_1^1$. However, the graph is non-deterministic. Transition from one relation to another, is dependent not just on the action, but also on various parameters like relative velocities, sizes, positions etc. The same action set (S, S) may also lead them to relations $\angle_1^0$ or $\angle_0^1$. Similarly, from relation $\angle_0^0$, the actions (S,P) or (M,P) may also result in the relation $\angle_0^5$ [DW07]. Unlike Fig.2.9 (where we consider only rotational motion), Fig.3.2 vessels have both translational and rotational motion.

Using the neighborhood transitions specified in Fig.3.2, we can build the transition graph of rule compliant behavior for each configuration. In this step-by-step transition graph, spatial transformations from a potential-collision state to a safe state can be determined.

3.1 Modelling Rule-Compliant Behavior

[DFW+07] describes rule-specific transition graphs which have the $OPRA_4$ relation between the vessels as nodes and rule-compliant actions as edges. The transition graph shown in Fig.3.3 is a transition sequence which specifies a prototype of the rule compliant actions from a potential collision state to a safe state. A typical configuration like *head-on* directs both the vessels to *turn starboard* . When they are not head-on anymore, they can go *midships*, and when they are just about side by side, they can *turn port*, heading back to their original course.(See Fig.3.3)

${}_4\angle_0^0$ is considered a potential collision state (*head-on*) and ${}_4\angle_4^4$ is a safe state (*side-by-side*). When two vessels have the relation ${}_4\angle_0^0$, the rule-compliant action is (S, S). The $OPRA_4$ relation after committing the specified action will be, ${}_4\angle_1^1$ as shown in Fig.3.3. It means, when two vessels have an $OPRA_4$ relation of ${}_4\angle_0^0$, and both vessels turn starboard, the succeeding relation is ${}_4\angle_1^1$. Similarly when the relation is ${}_4\angle_1^1$, and both vessels *keep midships* (don't alter direction), the succeeding relation is ${}_4\angle_2^2$. Since ${}_4\angle_2^2$ is an exact relation and unstable, it almost immediately becomes ${}_4\angle_3^3$. Finally, both vessels *turn port* (turn left), the two vessels are side-by-side and the relation is $(A_4\angle_4^4 B)$, a safe state.

But the transition graph in Fig.3.3 is an *idealized* transition sequence. Fig.3.4 gives the actual transition sequence that may take place. The colored nodes indicate the idealized thread for $OPRA_4$ transitions as represented in Fig.3.3. The start state of ${}_4\angle_0^0$ marks exact (or linear) regions. Since we consider the vessels to be $2D$ oriented points, exact relations

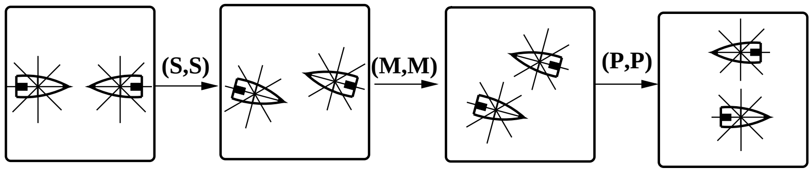


Figure 3.3: Idealized transitions for Head-on in $OPRA_4$: $A_4\angle_0^0 B \rightarrow A_4\angle_1^1 B \rightarrow A_4\angle_2^2 B \rightarrow A_4\angle_3^3 B \rightarrow A_4\angle_4^4 B$

are unlikely to occur and are unstable. This leads us to consider the neighboring relations of ${}_4\angle_0^0$, { ${}_4\angle_{15}^0$, ${}_4\angle_{15}^{15}$, ${}_4\angle_0^{15}$, ${}_4\angle_1^0$,${}_4\angle_0^1$ }, as the representation for *head-on*. Similarly, the safe-sate ${}_4\angle_4^4$ is also a exact relation and hence unstable. In practice we see its neighboring relations {${}_4\angle_3^4$, ${}_4\angle_3^4$, ${}_4\angle_5^5$}.

Assuming the velocity is same for both vessels, we expect that the resulting relation is at least a conceptual neighbor of the idealized relation. Depending on various parameters like position, velocity etc., of the vessels, the same action may lead to different relations with respect to the conceptual neighborhood graph. Hence, there is a necessity to incorporate the conceptual neighbors of each relation from the idealized thread.

The actual transition graph is better represented by Fig.3.4. The colored nodes represent the idealized transition sequence. Note that Fig.3.4 is only an approximation of the transition graph. For a more comprehensive graph, refer to [DFW+07].

To avoid the problem with exact relations like $\angle_0^0$, *k-OPAA* contains only interval relations. Let us consider 6-*OPAA*$[15, 45, 120, 240, 315, 345]$(Fig.3.1) for the same application. With *k-OPAA* the front of the object is not represented by a line but by a region. *Head-on* is represented by the *k-OPAA* relation $\angle_0^0$. and the safe state is $\angle_3^3$.

Fig.3.5 represents the idealized transition graph of *k-OPAA* relations for the *head-on* configuration. The unique advantage of using *k-OPAA* is that there are no unstable exact relations. Hence the idealized transition graph is the closer to the transitions actually seen in practice. *k-OPAA* also gives the advantage of having a fine granularity for the front and a coarser granularity for the back. The granularity of each direction can be chosen depending on the application.

We now use the step-by-step transition graphs to describe the rule compliant behavior of two vessels in a particular configuration. As we have identified in the previous section, $\angle_0^0$ and $\angle_3^3$ represent the potential collision and the safe states respectively. Fig.3.5

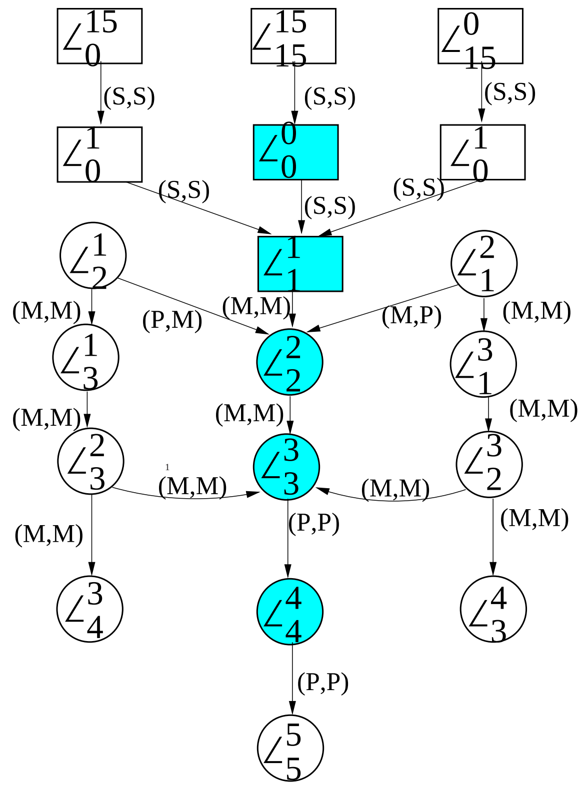


Figure 3.4: Actual Transition sequence of rule-compliant behavior for $Head - On$ using $OPRA_4$ (approximation from [DFW+07])

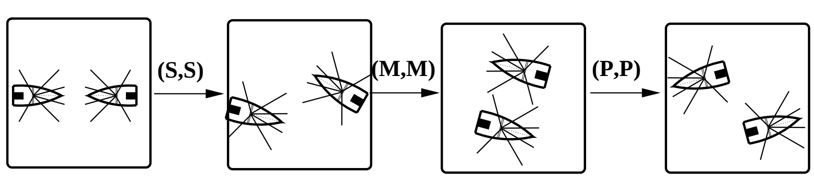


Figure 3.5: Idealized transitions for Head-on in 6-$OPAA[15, 45, 120, 240, 315, 345]$: $A\angle_0^0 B \rightarrow A\angle_1^1 B \rightarrow A\angle_2^2 B \rightarrow A\angle_3^3 B$

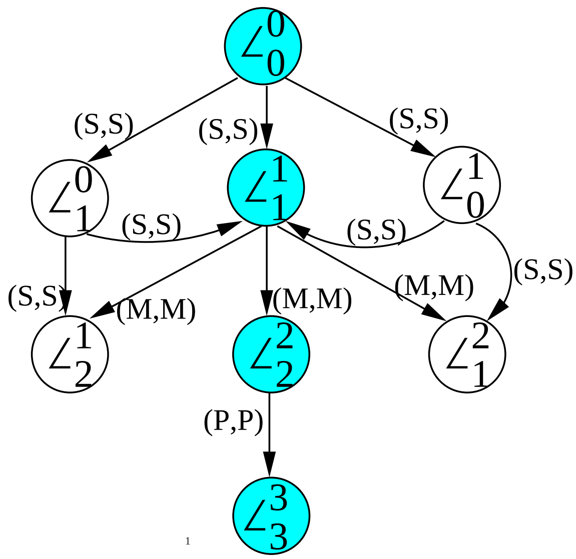


Figure 3.6: Actual Transition sequence of rule-compliant behavior for $Head - On$ using 6-$OPAA[5, 45, 120, 240, 315, 355]$

represents the idealized thread of the transition sequence. While the idealized thread is temporally complete, it is not a suitable formalization of all the possible relations that may occur while committing rule-compliant actions. As shown in Fig.3.2, the transition from one relation to another based on actions is non-deterministic.

The transition graph in Fig.3.6 is a more comprehensive picture. This graph shows the relations that might exist when two vessels exhibit rule compliant behavior for the *head-on* configuration. The colored nodes in the transition graph represent the ideal thread. For each of the neighborhood relations, we derive actions which will lead us back to the ideal thread to maintain rule-compliant behavior.

3.2 Software Architecture

See Fig. 3.7 for the basic architecture. In this project we have used three main components

- **Sailaway simulator**:- Supplies the exact positional information of the vessel in the simulator to Golog Decision Agent. The simulator also receives the commands from the Decision Agent.

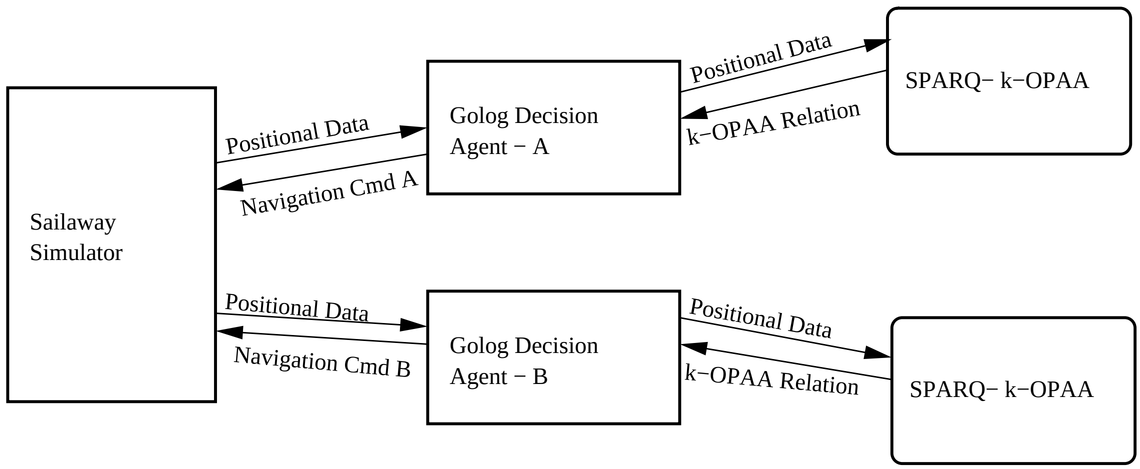


Figure 3.7: Architecture of Software

- **SPARQ** :- Converts the positional information into an *k-OPAA* binary relation and this information is sent back to Decision Agent.
- **Golog Decision Agent** :- Each vessel has its own Decision Agent. Using the *k-OPAA* binary relation the current spatial situation is mapped to a certain rule in the rule set and the corresponding decision is made. The decision is then supplied to the Sailaway simulator.

3.3 Golog Decision Agent

The Golog Decision Agent uses 6-*OPAA*$[15, 45, 120, 240, 315, 345]$ to map each situation to a certain configuration and then make the corresponding decision. The configuration *Head-On*, in 6-*OPAA* is represented by $\angle_0^0$. The relation $\angle_0^0$ is mapped to the corresponding configuration. The actions associated with this configuration will be implemented.

The decision agent is coded in Golog. The requisite code is:

```
proc(motor_kopaa(V1),
    [if([V2,Rel,Rel_P1,Rel_P2] = ecf_collissionWith(getOwnName),
        if(rule_kopaa(Rel_P1,Rel_P2,2,ecf_vesselType(V2)) = H,
```

```
                            /*head_on, rule H*/
5              send( V1, s), /* V1 turn starboard  */
               if(rule_kopaa(Rel_P1,Rel_P2,2,ecf_vesselType(V2)) = C1,
                            /*on left, rule C1*/
                  send( V1, m), /* V1 keep midships  */
                                      ......
```

There are different vessel types, Sport vessel, Motor vessel, Sail vessel etc. Each type of vessel has its own set of rules. This segment of code basically uses the 6-*OPAA* to decide whether the potential collision is on its front or left and takes the corresponding decision depending on its colliders' vessel type. Here we receive the information about the 6-*OPAA* as an exogenous fluent in line 2. An exogenous fluent is a sensor value which is taken from the external environment. In line 3, we test whether the given configuration falls under Rule H (head-on). If true, we ask the vessel to turn starboard. In line 6, we test whether the given configuration falls under Rule C1 (vessel approaching from left). If true, we ask the vessel to keep midships.

Chapter 4

Conclusion

Oriented points are an abstraction for describing objects which have an intrinsic front. We have presented a novel calculus, *k-OPAA*, for reasoning about the relative orientation using only angular intervals (sectors); the intent behind this is to enable cognitively attested tolerance intervals to be associated with exact relations, instead of being incorporated as extra-theoretical entities at implementation time as with other calculi. In order to determine the extent of some of these tolerances, a simple experiment was conducted on a group of 20 subjects.

We demonstrate that *k-OPAA* is a relation algebra in the sense of Tarski and defined the constraint satisfaction problem in this context. A tractable subset of the set of all relations was identified. We also looked at some example application of Qualitative Spatial Reasoning with *k-OPAA*, comparing it with other models. Also, in cognitive experiments, and also in implementations as in robot navigation, spatial regions are not crisp, but are graded (or fuzzy) at the boundaries. Nonetheless, the neat discretizations here, or in other JEPD calculi, may form a theoretically sound skeleton on which other relations can be defined. It would be important in future work to consider building the model (especially the transition algebra) in a probabilistic framework.

In the analysis here, no thought has been given to distance. In much spatial reasoning, distance and orientation are thought of as orthogonal[MR08] - so that the orientation relation remains unaltered with distance, but this is not the case; often the angular zones corresponding to similar degree of “frontness” may shrink as distance increases[MGNE00]. Also, at distances larger than a certain influence zone, the spatial relation is often not perceived as being “Front”, etc., indeed here also there may be some asymmetry between different orientations like “front” and “back”.

Another difficulty with incorporating distance into such a calculus is that the notion of qualitative distance is often a function of object size, which is abstracted away in the point object assumption.

In terms of applications of spatial calculii, we feel *k-OPAA* is perhaps somewhat better suited to real life situations than other calculi owing to its explicit provision for tolerancing, and also the handling of asymmetric angular sectors.

Among other theoretical aspects, we have not identified the maximal tractable subset for the calculus. Some algorithms to find the tractable subsets have been provided by [Ren07]. Another area of exploration may be to consider introducing a temporal dimension to *k-OPAA* - i.e. to consider the transition sequences as atomic events, and considering how these would build up to larger processes, perhaps with associated probabilities.

Bibliography

[All83] James F. Allen. Maintaining knowledge about temporal intervals. *Commun. ACM*, 26(11):832–843, 1983.

[CH01] A G Cohn and S M Hazarika. Qualitative spatial representation and reasoning: An overview. *Fundamenta Informaticae*, 46(1-2):1–29, 2001.

[DFW+07] Frank Dylla, Lutz Frommberger, Jan Oliver Wallgrün, Diedrich Wolter, Stefan Wölfl, and Berhard Nebel. Sailaway: Formalizing navigation rules. In *Proceedings of the AISB'07 Artificial and Ambient Intelligence Symposium on Spatial Reasoning and Communication*, 2007.

[DM05] Frank Dylla and Reinhard Moratz. Exploiting qualitative spatial neighborhoods in the situation calculus. In *Spatial Cognition IV Reasoning, Action, Interaction*, volume 3343 of *Lecture Notes in Artificial Intelligence*, pages 304 – 322. Springer; Berlin; http://www.springer.de, 2005.

[DW07] Frank Dylla and Jan Oliver Wallgrün. Qualitative spatial reasoning with conceptual neighborhoods for agent control. *J. Intell. Robotics Syst.*, 48(1):55–78, 2007.

[FLWD07] Lutz Frommberger, Jae Hee Lee, Jan Oliver Wallgrün, and Frank Dylla. Composition in opram. Technical Report 013-02/2007, SFB/TR 8 Spatial Cognition; http://www.sfbtr8.uni-bremen.de/, 2007.

[Fra91] A.U. Frank. Qualitative spatial reasoning about cardinal directions. *Auto-Carto 10*, 1991.

[Fre91] C. Freksa. *Conceptual neighborhood and its role in temporal and spatial reasoning*. Inst. für Informatik, 1991.

[Gap94] K.P. Gapp. Basic meanings of spatial relations: computation and evaluation in 3D space. *Proceedings of the twelfth national conference on Artificial intelligence (vol. 2) table of contents*, pages 1393–1398, 1994.

[Gua99] Coast Guard. *Navigation rules: International-Inland* . United States Dept. of Transportation, 1999.

[JT93] B. Jónsson and C. Tsinakis. Relation algebras as residuated Boolean algebras. *Algebra Universalis*, 30(4):469–478, 1993.

[KDK+04] Alexander Klippel, Carsten Dewey, Markus Knauff, Kai-Florian Richter, Dan R. Montello, Christian Freksa, and Esther-Anna Loeliger. Direction concepts in wayfinding assistance systems. In Jörg Baus, Christian Kray, and Robert Porzel, editors, *Workshop on Artificial Intelligence in Mobile Systems (AIMS'04)*, pages 1–8. SFB 378 Memo 84; Saarbrücken, 2004.

[Lan03] B. Landau. Axes and direction in spatial language and spatial cognition. In E. vanderZee and J. Slack, editors, *Representing direction in language and space*. Oxford: Oxford University Press, 2003.

[Lev03] S. Levinson. *Space in language and cognition : Eplorations in cognitive diversity*. Cambridge University Press, 2003. Series Title: Language Culture and Cognition, 5.

[LM94] P.B. Ladkin and R.D. Maddux. On binary constraint problems. *Journal of the ACM (JACM)*, 41(3):435–469, 1994.

[LR04] G. Ligozat and J. Renz. What is a qualitative calculus? A general framework. *PRICAI 2004: Trends in Artificial Intelligence: 8th Pacific Rim International Conference on Artificial Intelligence, Auckland, New Zealand, August 9-13, 2004: Proceedings*, 2004.

[Mac77] A.K. Mackworth. Consistency in networks ofrelations. *Artificial Intelligence*, 8(1):99–118, 1977.

[Mad06] R.D. Maddux. *Relation Algebras*. Elsevier Science, 2006.

[Mal06] Elbert S. Maloney. *Chapman piloting and seamanship 65th Edition (Chapman piloting, Seamanship and small boat handling)* . Hearst, 2006.

[MDF05] R. Moratz, F. Dylla, and L. Frommberger. A relative orientation algebra with adjustable granularity. *Proceedings of the Workshop on Agents in Real-Time and Dynamic Environments (IJCAI 05)*, 2005.

[MGNE00] A. Mukerjee, K. Gupta, S. Nautiyal, and Et. Conceptual description of visual scenes from linguistic models. *Image and Vision Computing*, 18:173–187, 2000.

[MR08] R. Moratz and M. Ragni. Qualitative spatial reasoning about relative point position. *Journal of Visual Languages and Computing*, 19(1):75–98, 2008.

[MRW00] R. Moratz, J. Renz, and D. Wolter. Qualitative spatial reasoning about line segments. *Proceedings of the Fourteenth European Conference on Artificial Intelligence. Wiley*, pages 234–238, 2000.

[Ren07] Jochen Renz. Qualitative spatial and temporal reasoning: efficient algorithms for everyone. In Manuela M. Veloso, editor, *IJCAI*, pages 526–531, 2007.

[RM04] J. Renz and D. Mitra. Qualitative direction calculi with arbitrary granularity. *PRICAI 2004: Trends in Artificial Intelligence: 8th Pacific Rim International Conference on Artificial Intelligence, Auckland, New Zealand, August 9-13, 2004: Proceedings*, 2004.

[RN99] J. Renz and B. Nebel. On the complexity of qualitative spatial reasoning: A maximal tractable fragment of the Region Connection Calculus. *Artificial Intelligence*, 108(1-2):69–123, 1999.

[RS88] G. Retz-Schmidt. Various views on spatial prepositions. *AI Magazine*, 9(2):95–105, 1988.

[VSF+97] C. Vorwerg, G. Socher, T. Fuhr, G. Sagerer, and G. Rickheit. Projective relations for 3D Space: Computational model, application, and psychological evaluation. *AAAI/IAAI*, pages 159–164, 1997.